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The Use of Artificial Stock Markets in Financial Research

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Declaration

Prohlašuji, že jsem svoji bakalářskou práci napsal samostatně a použil pouze uvedené zdroje.
I hereby declare that I have elaborated this thesis on my own and that I have used only the sources listed.

Prague, date ______________ ____________________

signature

Abstrakt

Tato práce ukazuje využití nové metodologie multiagentních systémů (agent-based modelling) ve výzkumu teoretických financí. V první části je vyložena tato metodologie a jsou v ní představeny některé modely, které tohoto přístupu využívají. V další části je představen model umělého akciového trhu Genoa Artificial Stock Market, jenž je implementován v softwaru Matlab. Některé vlastnosti tohoto modelu se zaměřením na stylizovaná fakta finančních trhů jsou prezentovány. Dále jsou navrhnuty a implementovány dvě úpravy, které s využitím poznatků z behaviorálních financí umožní vytvořit provázanost mezi výnosy jednotlivých akcií a mohou být jedním z vysvětlení finančních krizí. V závěru jsou výsledky shrnuty a dále jsou navrhnuta další řešení, která by mohla model rozšířit a učinit jej realističtějším.

Klíčová slova: agent-based výpočetní modely, komplexní systémy, umělé akciové trhy, stylizovaná fakta finančních trhů

Abstract

This thesis deals with applications of a new methodology of agent-based modelling on research in finance. In the first part, the methodology of agent-based computational finance is explained and some models, which use this approach, are presented. In the next part, a model of an artificial market, the Genoa Artificial Stock Market, is introduced. We implement it in Matlab software, which enables us to assess properties of the market with emphasis on stylized facts of financial markets. Furthermore, two modifications of the model are proposed and implemented, which employ some knowledge from behavioural finance. These modifications create interdependence between returns of shares and can be one of possible explanations of financial crises. We conclude by summarizing the results and suggesting further modifications that could enhance the model and make it more realistic.

Keywords: agent-based computational finance, agent-based computational economics, complex systems, artificial stock markets, stylized facts of financial markets

Acknowledgements

I want to express my gratitude to my supervisor Petr Švarc for his helpfulness, inspiration, initiating me into the study of this field of economics and for his eternal support.

"Levy, Solomon and Levy's Microscopic Simulation of Financial Markets points us towards the future of financial economics. If we restrict ourselves to models which can be solved analytically, we will be modeling for our mutual entertainment, not to maximize explanatory or predictive power."

Harry M. Markowitz, President, Harry Markowitz Co.,
and Nobel Laureate in Economics

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Chapter 1

Introduction

Financial markets have been one of the most complex areas of interest of social sciences in the last decades. Their importance has grown since more and more money flows through them and several exotic tradeable assets have been invented. Huge amounts of money flow through the markets every day. In January 2009, 1.066 billion trades in shares in the value of USD 5.814 trillion were accomplished only in the member exchanges of the World Federation of Exchanges[1]. Money flows through foreign exchange or derivatives markets are much higher.

Financial markets have also been subject to regulations, various controls and interest of the media and as a result, enormous amount of data is available. Financial researchers and practitioners have been investigating these data and developing new theories, trading strategies, sophisticated instruments etc. Although the research has been successful in many areas, several puzzles that have not been satisfactorily tackled remain in existence.

Several research methodologies have emerged as a response to failure to find analytically tractable models which would at least replicate some of the puzzles. Some of these approaches are connected with physics - just the names, such as financial engineering or econophysics, allude to the fact that some methods of physics are in use.

The main aim of this thesis is to introduce agent-based computational finance (ACF), another important approach to model various areas in finance. In the first part of the thesis, we explain the basics of agent-based computational economics and its applications in finance. We give some examples of existing models that employ this

[1] World Federation of Exchanges, accessed 21 April 2009,
<http://www.world-exchanges.org/statistics/ytd-monthly>

methodology.

Next, we introduce an existing model called the Genoa Artificial Stock Market. We investigate its properties with an emphasis on stylized facts of financial markets with the use of our implementation of the model in Matlab[2]. In simple cases when only irrational investors trade stocks in the market, the model provides good results that are consistent with reality. However, as more rational traders are added to the model, some unfavourable properties of time series of returns emerge. We try to reduce these drawbacks by an introduction of endogeneous risk aversion of investors, which makes use of some elements of behavioural finance. In addition, it enables us to create a form of interdependence of asset returns observed in reality, that can serve as one possible explanation of financial crises. Finally, the obtained results are summarized and some proposals are introduced that could improve the model and make it more realistic.

[2]Scripts were written in the Matlab software installed on computers at the Faculty of Social Sciences of Charles University in Prague. Copies of the scripts are available on the included CD.

Chapter 2

On agent-based modelling in finance

In this chapter, we introduce agent based computational finance, a methodology that we have decided to use in this thesis. We also provide the reader with examples of four models that employ this methodology. Finally, we present stylized facts of financial markets, a phenomena that analytical models are not able to replicate or explain but ACF models are quite successful in doing this.

Agent-based computational finance (ACF) is an application of agent-based computational economics (ACE) methodology on financial phenomena. ACE models economic issues as complex adaptive systems (Tesfatsion (2006)). The systems are formed by large amount of heterogeneous agents, whose behaviour evolves as a response to changing environment. The environment is formed by behaviour of agents, so a feedback loop exists and it is impossible in most cases to solve the models without use of a computer. It is important to note that even with the use of a computer, no solution of the model usually exists. The purpose of agent-based models is not to find a stable equilibrium but to study a process how an equilibrium is reached (if it exists) or how investigated variables behave. We have not mentioned yet what an agent is. An agent is an entity characterized by data (or properties) and methods (this is strikingly similar to objects in object oriented programming but agents are built to be active parts in a model, whereas objects are programmed to be called by external participants). Examples of agents can be individuals, such as customers, employees, traders or groups of individuals, such as firms (composed of owners, managers, employees), governments, institutions or non-living entities (goods, weather) etc.

Another characteristic of a complex dynamic system is worth mentioning. The result of agents' behaviour on a macro level is not the sum of behaviours of the individual agents. Sometimes a very complex macrobehavior emerges from microbehaviour and it is usually unpredictable even if we know how every single individual agent behaves (this phenomenon is called emergence). An excellent example of emergence is Schelling's segregation model[1]. He showed that total segregation of neighbours of different characteristics can result as a consequence of small preference towards one type of characteristic.

Although some ACE models have been used for practical purposes (to predict future asset price movements etc.), most of their importance lies in theoretical research. L. Tesfatsion recognizes four objectives of research in ACE:

- Empirical understanding: the models can answer why some regularities have emerged from individual heterogeneous behaviour without centrally planned incentives. They can also answer the question about what conditions are necessary and what conditions are unnecessary in order for the regularities to emerge.

- Normative understanding: the major drawback of economic research is that it is usually impossible to perform controlled experiments. This lack can be partly resolved by ACE, where well-constructed agent-based models can suggest possible impacts of proposed policies (L. Tesfatsion draws a parallel between the models and laboratories).

- Qualitative insight and theory generation: to answer not only why some regularities have evolved but also to answer why other have not evolved. This is usually done by changing initial conditions and observing the evolution of the system.

- Methodological advancement: ACE is relatively new methodology and no unified procedures of research exist. Every single project is a sort of an art. One of the aims of research in ACE is to provide new researchers with techniques that would make an entry into this field as simple and attractive as possible. As economists are usually not very accustomed to software development, several platforms for simulations in social sciences have been implemented. In addition, ACE researches seek to discover simple ways to test hypothesis in models, present results of simulations etc.

Agent based computational finance, as a field of ACE, has had several topics of interest, which are summarized in an excellent study by LeBaron (2006). In this thesis, we are going to deal with stylized facts of financial markets, which have not been

[1]Schelling, T. C. (1971).Dynamic models of segregation.*Journal of Mathematical Sociology*,1,143-186.

explained by any analytical model. More on the stylized facts will be stated in the next section, along with the tests for their presence in financial time series.

ACF models traders as heterogeneous boundedly rational individuals, whose behaviour evolves through time as a result of a learning process. This is in contrast with perfectly rational representative agent. Agents can be further "improved" by incorporating some aspects known from behavioural finance, such as loss aversion, overconfidence, overreaction etc. This all is in accord with new paradigm of adaptive markets proposed in Lo (2004).

Several agent based models of financial issues or whole markets have been developed. Some of them are described in already mentioned study by LeBaron (2006). Some other can be found in Levy *et al.* (2000). We will mention only four models.

Although agent-based models in finance might seem obscure, modern and unconventional, several prominent authors in economics and finance have been among the pioneers who built some of the first models. In Stigler (1964), the author investigated conditions under which a stock market (New York Stock Exchange, specifically) works effectively. The founder of portfolio theory and another laureate of the Nobel Prize, Harry Markowitz, is yet a co-author of a simulation programming language SIMSCRIPT. In an article with Kim[2], they investigate reasons of the 1987 stock market crash. According to their simulations, the crash was caused endogenously by behaviour of traders. When a loss occurred, they started to sell shares to prevent further losses and a large slump in prices followed. This slump was exacerbated by computerized automated trading, which was followed by multitude of traders. According to the model, a large amount of traders who insure their portfolios in this fashion destabilizes markets and causes prices to change extremely quite often.

Probably the best known artificial stock market is the *Santa Fe Artificial Stock Market* (SFI-ASM). The first version was developed in the Santa Fe Institute in the late 1980s and the early 1990s. Its inventors, Brian Arthur and John Holland, had planned to build an artificial stock market with two requirements. Firstly, they wanted to build a market in which trading strategies would live as an ecosystem. Better strategies would survive and would be further improved and worse strategies would vanish. The second requirement was to make a market as simple as possible. In addition, only few parameters would be specified at the beginning and the market would "live" and evolve on its own without external interventions.

[2]Kim, G., Markowitz, H.M. (1989).Investment rules, margin, and market volatility.*Journal of Portfolio Management*.

The first requirements was fulfilled with utilization of genetic algorithms and classifier systems, both of whom were invented by John Holland. The second requirement was met by making some very simplifying assumptions but still a great scope for research was left.

The structure of the SFI-ASM is very simple. Only one risky asset (shares of a company) and one risk-less asset (a bond) are traded on the market. The share pays stochastic, mean reverting dividends and the bond pays a fixed interest rate. The weakest point in the original paper by Palmer *et al.* (1994) was its price determining mechanism. Agents issued their orders to buy, sell or keep assets according to rules that evolved through time. Price adjusted very simply according to excess supply or excess demand:

$$p_{t+1} = p_t + \alpha(D(p_t) - S(p_t))$$

This excess was then randomly cancelled and the remaining market orders were fulfilled. An apparent drawback of this approach was high sensitivity of price adjustment to parameter α. In the second version published in Arthur *et al.* (1997), price of the stock moved according to expectations that traders formed on the basis of their forecasting rules.

The most original innovation in this market is the evolution of trading and forecasting rules. Each agent acts according to 60 rules and these rules are different and independent among all traders. Each rule consists of 3 parts. The first, condition part, describes conditions on the market on which a rule is activated. Fundamental and technical conditions can be taken into account. A condition, such as "the 20 period moving average of price is below the 100 period MA of price" is an example of technical condition. An example of fundamental condition might be: "the dividend went up two periods ago". The second part of a rule is an action part - if a rule is activated, the action part makes a trader buy or sell a share. The final part is the strength of a rule. It summarizes how the rule has been successful in increasing trader's wealth so far. This has been a short summary of a classifier system.

The condition parts of rules are encapsulated in a string consisting of characters of ternary alphabet: {0,1,*}. The length of the string depends on how many characteristics of a market can be recognized. The authors have usually chosen between 70 and 80 letters in a a string. A market condition is then translated in a string, which consists of zeros and ones. These strings are then compared with condition parts of the rules, where '*' means 'do not care'. So, a string of length of 4 characters "1101" can activate rules, whose condition parts are "*1*1", "1101", "110*" etc. A rule is finally activated with probability proportional to its strength. If no rule can be activated, a trader does not issue any order.

The rules are not static, they evolve according to a genetic algorithm, which replaces 10 - 20% of rules on average. Several operations known from biology can occur. A crossover combines two parts of a string, each one from a different "parrent" in order to produce an "offspring" and a mutation replaces random parts of strings.

As Levy *et al.* (2000) points out, results of simulations in SFI-ASM differ in two cases. In the first case, relatively few traders operate on the market and changes in dividends are not very large. In this case, price of the stock approaches an equilibrium price which is not very distant from the fundamental price. Only small volumes of assets are traded, no bubbles or crashes (large deviations from fundamental price) or price anomalies come up. Furthermore, relatively small amount of trading rules survives.

In the second case, a large amount of traders trade in the market and conditions are more general (i.e. large or small changes in dividends arise). In this case, price does not tend towards some equilibrium. Furthermore, it often makes sudden and large moves and bubbles and crashes are not rare events. In addition, there is a substantial correlation between occurrence of bubbles and crashes and trading volumes (during these events, larger volumes are traded). Finally, wider variety of trading rules evolves.

Another brilliant example of an artificial stock market is the *Genoa Artificial Stock Market (GASM)*, which has served as the main source for this thesis. Unlike SFI-AFM, where only one asset is traded, this market is more general and unlimited amount of assets can be traded by unlimited amount of heterogeneous traders. The traders are heterogeneous in two dimensions. In the first one, they belong to various categories, which are characterized by unique trading strategy. In the second dimension, traders are heterogeneous even across the different categories. That means that a trader can belong for example to category of mean-variance traders, i.e. he or she trades on the basis of portfolio optimization rule, which compares means and covariance matrices of returns of assets. In addition, the trader is heterogeneous in the sense that he or she uses different parameters in the optimization rule (such as risk aversion or a period with which they analyse the past data). Of course, agents are heterogeneous by the amount of cash and assets that they possess.

Several modifications of the GASM have been introduced but all of them have several features in common. A clearing mechanism, which is a simple version of the limit order market, is used. More on this is discussed in one of the later sections. Large amount of traders is present, each of whom is endowed with finite cash and finite amount of assets at the beginning of simulations. The traders cannot buy assets with borrowed funds (leverage purchases are not possible) and they are not allowed to sell assets that they do not own (short selling is forbidden). One more important fact is worth mentioning - in all the models, no fundamental variables, such as dividends, affect the behaviour of traders. Strategies of traders take into account only the past movements of prices. This drawback should be taken into account in further research but we feel that simpler versions here presented should be firstly improved.

In Raberto *et al.* (2001), shares of a single company are traded by zero-intelligence agents who issue random orders. The model presented in this paper reproduces successfully leptokurtic probability distribution and volatility clustering of returns. Trading with more assets was introduced in Cincotti *et al.* (2005). The assets are still traded by zero-intelligence agents who buy or sell assets randomly and decide on the proportion of risky assets on their total wealth also at random. Two sets of simulations have been run in this paper. In the first one, the shares do not pay any dividend. In this case, volatility clustering and fat tails are replicated but hypothesis that shares follow random walk is rejected. This is caused by the finiteness of financial resources that causes shares to follow a mean reverting process. In the second sets of simulations, shares pay stochastic dividends and traders buy shares again randomly. Time series generated in the simulations have the same favourable properties as in the first case but in addition, share prices follow random walk. However, the path of stock price is not still very satisfactory, since the payments of dividends only inflate stock prices steadily and no crashes occur as in the real world.

Three new categories of traders have been introduced in Cincotti *et al.* (2003). In this paper, the authors investigate how wealth is redistributed among agents pursuing different trading strategies (the strategies will be described later on). This redistribution differs in case when no cash inflow is present (in this case, share prices revert to the mean again) and case when a cash inflow in the form of paid dividends is present. Another influence is given by length of periods on which agents base their portfolio decisions. A drawback of this paper is that unacceptable disproportion between the number of zero intelligence traders (10,000) and other types of traders (10 in each category) was chosen. In this thesis, we will demonstrate what features change as a result of different market set-ups, i.e. when we add more traders of the latter categories.

2.1 Stylized facts of financial markets

Financial time series are characterized by various anomalies known as stylized facts of financial markets. The most of what is known about them comes from research that tried to validate or disprove the efficient market hypothesis. Our aim is not to depict all stylized facts but only those that can be replicated by the agent-based model presented in this thesis. In this section, we shall describe the stylized facts that will be tested on data from our simulations. Furthermore, we shall describe statistical tests that check for the presence of these stylized facts in financial time series. Finally, we shall show how the tests work on real data of Standard and Poors' 500 Index.

Our descriptions of stylized facts draw on a detailed study Cont (2001), Cont (2004) some also on Ehrentreich (2008) and finally Jondeau *et al.* (2007). The tests are overtaken from the last two cited works and from Alexander (2001).

Tests in this sections are demonstrated on real data of Standard and Poor's 500 Index between January 1 1970 and March 27th 2009. The index's daily values and daily returns are plotted in figure 2.1. Several important dates are marked on the x-axis. These dates mark periods of beginnings of financial turmoils or stock market crashes. In the following concise overview of these events, we draw upon Dow (2009).

11th January 1973 is regarded as the beginning of the 1973-1974 bear market. This two years slump had several causes. It came in shaky times after the fall of the Bretton-Woods system and the devaluation of the dollar brought about by the Smithonian agreement. Another reason for the downturn was rising inflation, exacerbated by the first oil shock of 1973. Measures to curb the inflation, such as price controls on wages, oil prices and general goods did not help the stock markets, as well as political instability, triggered by the Watergate scandal. S&P index fell 45% between January 11 1973 and December 10 1974, which classified the decline of this period among the worst in history. **The 19th October 1987** is the date of the infamous Black Monday. The crash came during a several days long bear market and it caused the largest one-day decline in the history of the S&P index reaching 22.6%. **August 1998** was another period of the market decline. The primary cause was the Russian financial crisis, during which Russia announced government defaults on their debt. It caused severe troubles mainly for the huge Long Term Capital Management mutual fund, which was finally bailed out by the New York Fed for several billion dollars. **10th March 2000** is the date when the dot com bubble burst. This date sparked a meltdown in financial markets, whose mildly recovery was interrupted by the 9/11 terrorists and the markets did not fully recover until October 2002. On **September 29 2008**, the

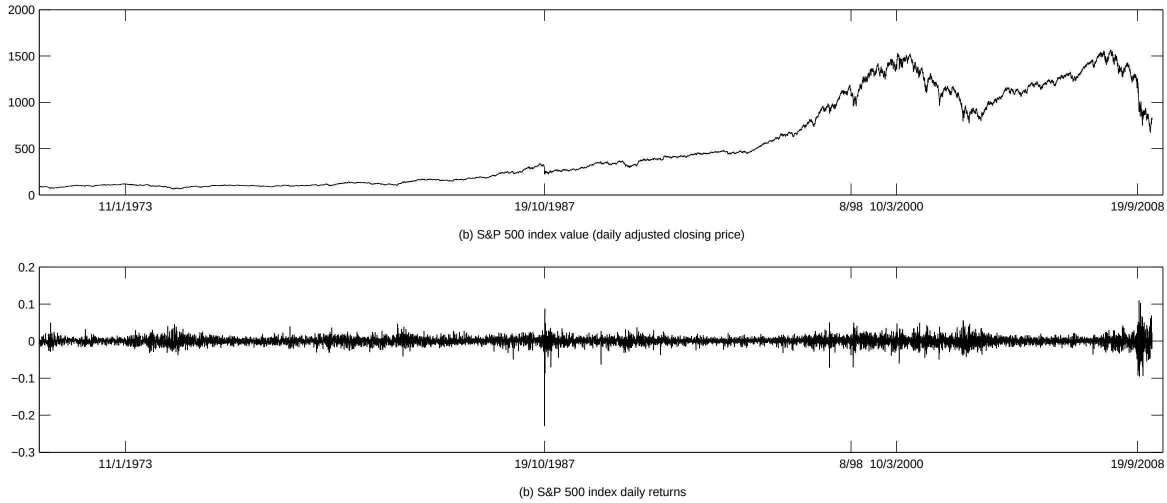


Figure 2.1: Standard & Poor's 500 index daily values and returns between January 2 1970 and March 27 2008;
source: http://finance.yahoo.com/

White House's plan to rescue the American economy amongst the financial crisis was rejected by the House of Representatives. It reinforced fears in the markets that a long recession might eventually occur without the bailout programme.

We can see from Figure 2.1 that the volatility of returns was usually higher following these days. This is in example of volatility clustering, one of the stylized facts that we are now going to present.

2.1.1 Absence of autocorrelations

The first stylized fact is in line with efficient market hypothesis. It states that autocorrelation functions of returns are usually insignificant, which implies that returns are unpredictable in practice. This does not hold usually for intraday returns (e.g. returns during 20 minutes long intervals), where market micro structure plays a role. Also autocorrelation functions of longer term returns, such as weekly or monthly returns, are sometimes significant, as Cont (2001) points out.

We will use **Ljung Box Q test**[3], sometimes known as a portmanteau test, to test for the significance of autocorrelation function of returns. Autocorrelation function is a function that maps lag j into autocorrelation of a random process with lag j.

[3] Ljung, G. & Box, G. On a measure of lack of fit in time series models.*Biometrika*.Biometrika Trust, 1978, 65, 297-303

Autocorrelation with lag j is defined as

$$\rho_j = \frac{\gamma_j}{var(r_t)} = \frac{\gamma_j}{\gamma_0}, \gamma_j = cov(r_t, r_{t-j}), 0 \leq j < T - 1.$$

Its sample counterpart is defined as

$$\hat{\rho}_j = \frac{\sum_{t=j+1}^{T} (r_t - \bar{r})(r_{t-j} - \bar{r})}{\sum_t^T (r_t - \bar{r})^2}.$$

The null hypothesis in the Ljung-Box Q test is that all autocorrelation of lags up to p are zero. The alternative hypothesis is that at least one of the autocorrelations is different from zero.

To test for the null hypothesis, Ljung-Box Q statistic is constructed:

$$Q_p = T(T+2) \sum_{j=1}^{p} \frac{1}{T-j} \hat{\rho}_j^2$$

It can be shown that $Q_p \sim \chi^2(p)$ under the null hypothesis. We have tested time series for 20 lags at the 0.05 significance level (as the authors of Matlab documentation do).

2.1.2 Fat tails of return distributions and their aggregated normality

At the outset of research in finance, it was widely believed that returns on assets are normally distributed. This conjecture was challenged firstly by Mandelbrot (1963), who showed that the distribution of returns on cotton prices depart from the normal distribution. Similar tests were performed later and it was shown that distributions on asset returns are usually leptokurtic (they have fat, heavy tails). It follows that normal distribution underestimates an occurrence of booms or recessions. Financial time series exhibit excess kurtosis for short-period returns and as the length of a period increases, returns return back to normality.

Kurtosis is defined as a standardized fourth centered moment of random variable X:

$$\kappa = \frac{E(X-\mu)^4}{\sigma^4},$$

where μ is the expected value of X and σ is the standard deviation of X.

An unbiased estimator of kurtosis is:

$$\kappa = \frac{1}{N-1} \frac{\sum_{i=1}^{N} (x - \bar{x})^4}{s^4},$$

where s^2 is an estimate of the variance of X and $\bar{x}$ is the sample mean of X. Sometimes, excess kurtosis is used instead of simple kurtosis. It measures the deviation of kurtosis from the kurtosis of the normal distribution, which is 3. So the excess kurtosis is defined as $\kappa' = \kappa - 3$. If the excess kurtosis is positive, the distribution of variable X is called leptokurtic and fat tails are present. On the other hand, if the excess kurtosis is lower than 3, the distribution is platykurtic and it has thinner tails.

In addition to tests on kurtosis, we will test our time series on normality. Jarque Bera test of normality of distribution is a test based on moments of distribution. It utilizes the fact that normal distribution has both skewness and excess kurtosis equal to zero. It has been shown[4] that the Jarque Bera test statistic is asymptotically distributed as chi-sqare with 2 degrees of freedom:

$$JB = T\left[\frac{\hat{S}}{6} + \frac{(\hat{\kappa}-3)^2}{24}\right] \sim \chi^2(2),$$

where T is the sample size and $\hat{S}$ is an unbiased estimator of skewness defined as:

$$\hat{S} = \frac{\sqrt{n(n-1)}}{n-2} \frac{\frac{1}{n}\sum(x_i-\bar{x})^3}{\left[\frac{1}{n}\sum(x_i-\bar{x})^2\right]^{\frac{3}{2}}}$$

2.1.3 Volatility clustering and long memory property of asset returns

It is easy to see from the Figure 2.1 that volatility (standard deviation) of returns is not constant for various time spans. That is, periods of tranquillity and volatility alter through time. Furthermore, as Mandelbrot[5] states "large changes tend to be followed by large changes - of either sign - and small changes tend to be followed by small changes". These two facts are signs of volatility clustering. Volatility clustering has led to a research on volatility from which currently widely used models, such as ARCH and GARCH have resulted. Owing to the volatility clustering, size of returns on financial assets can be forecasted. However, the sign of the returns is still unknown. Volatility clustering might also be one of the reasons of fat tails, since volatility persistent processes generate fat tails.

[4]Jarque, Carlos M.; Anil K. Bera (1980). "Efficient tests for normality, homoscedasticity and serial independence of regression residuals". *Economics Letters* 6 (3): 255–259.

[5]Mandelbrot, B. The variation of certain speculative prices Journal of business, UChicago Press, 1963, 36, 394

Two variables are usually taken as proxies for volatility. The first one are absolute returns $|r_t|$, the second one squared returns on assets. Both of these proxies tend to have significant autocorrelation functions for various lags. Two tests are usually employed to test for volatility clustering. The first one is an application of Ljung Box Q test on squared (absolute) returns. If the null hypothesis (no serial correlation) is rejected, volatility clustering is present in the data.

The second test for volatility clustering, which we will use, is the so called ARCH test proposed by Engle. It test for the presence of ARCH effect (autoregressive conditional heteroskedasticity in residuals). The test proceeds in 4 steps:

1. Mean of returns is deducted from returns and the residual is saved: $\hat{u}_t = ret_t - \bar{ret}$.

2. A regression of squared residuals on their own q lags is run and R^2 from the regression is obtained: $\hat{u}_t^2 = \gamma_0 + \gamma_1 \hat{u}_{t-1}^2 + \gamma_2 \hat{u}_{t-2}^2 + \ldots + \gamma_q \hat{u}_{t-q}^2 + v_t$, where v_t is an error term.

3. The test statistic defined TR^2, where T is the number of observations, is distributed as $\chi^2(q)$.

4. The null hypothesis is that no ARCH effect is present, which is equivalent with hypothesis that there is no volatility clustering. In this case, it holds that $\gamma_1 = \gamma_2 = \ldots = \gamma_q = 0$. An alternative hypothesis, that volatility clustering is present, means that at least for one $i, i = 1, 2, \ldots, q$ holds, that $\gamma_i \neq 0$.

As Ding *et al.* (1993) point out, autocorrelations of absolute and squared returns are significant for very long lags. Furthermore, power transformations of absolute returns $|r_t|^d, d > 0$ are also significant for long lags. In addition, they investigate several properties. The highest autocorrelations exist for $d = 1$ and for a given lag, they are decreasing with a distance of d from 1. They also state that autocorrelations of absolute returns are higher than autocorrelations of squared returns for long lags (usually for up to 100 lags). These characteristics indicate that returns on stocks have 'long memory' and they are not independent, although they are usually not autocorrelated (as suggested by the Efficient market hypothesis). The explanation of dependence of returns is in the fact that as information arrive, stock prices change accordingly and future returns take the old information into account (although their returns are not correlated).

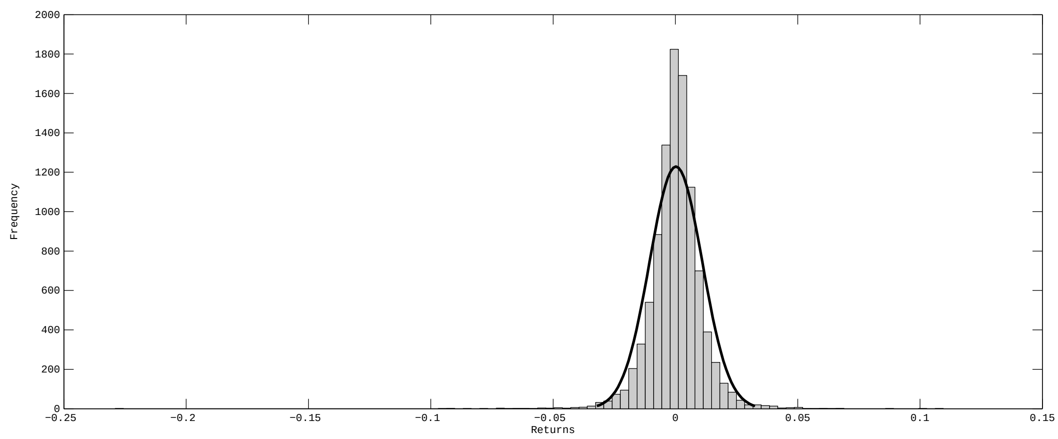


Figure 2.2: Histogram of the Standard & Poor's 500 index daily returns

2.1.4 Stylized facts in Standard & Poors' Index

We shall illustrate the mentioned stylized facts on real data of Standard & Poors' Index. The statistics can be found in Table 2.1.

It can be easily seen from histogram of returns (Figure 2.2) that the distribution of returns is leptokurtic (it is characterized by fat tails). Returns quite faraway from a fitted normal distribution arised, although probability of their occurrence is nearly zero. That is, normal distribution underestimates probabilities of crashes or extreme booms. Presence of normality of returns was rejected also formally using Jarque Bera test, whose test statistic is much higher than the critical value.

The stylized fact about no serial correlation in raw returns does not strictly hold for the S&P index. The same conclusion was proposed by Ding *et al.* (1993), who examined the index for a different time span. The autocorrelation of order 1 is significant which indicates that some proportion of prices of shares that comprise the S&P index might be predictable (which is in contrast with the efficient market hypothesis). The autocorrelation of order 2 is again significant but negative, which suggest that share prices process is mean reverting. Several other autocorrelations at various lags are also significant. This is supported by the Ljung Box Q test, which rejects the null hypothesis of no serial correlation in returns.

Absolute values of autocorrelations of squared returns are positive, significant at long lags and decreasing in lags which indicates long memory of absolute returns and volatility clustering of returns. The first lag at which the autocorrelation is insignificant is 328, which is in accord with hypothesis that returns have long-term memory. Volatility clustering is accepted also by the ARCH test, whose statistic is well above the critical value.

Number of observations	9904
Min	-0.229
Max	0.1096
Mean	0.00022
Std. dev.	0.0108
Studentized range	31.35185185
Skewness	-1.107
Kurtosis	31.7197
JB stat.	342400
Critical value (5% sign. level)	5.9817
LBQ stat.	89.831
Critical value (5% sign. level)	31.4104
ARCH stat.	185.1053
Critical value (5% sign. level)	3.8415

Table 2.1: Descriptive statistics of returns on the Standard & Poor's index

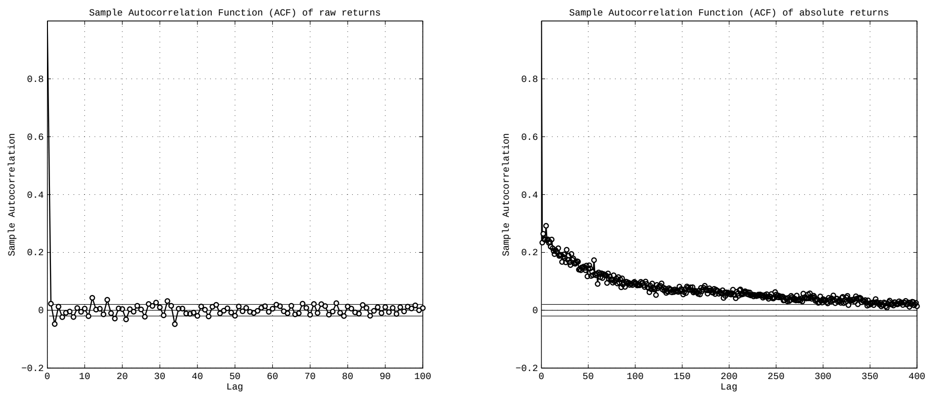


Figure 2.3: Autocorrelation functions of raw and absolute returns on the Standard & Poor's index

Chapter 3

The Model

In this chapter we show the building blocks of the model. In the first part, we provide an overview of the model. Next, we show how an important part, price determination works. Various types of traders are described in the subsequent section and the final section concludes with a description of how an adjustment of endogenous risk works.

3.1 An overview of the model

The central framework of the model is very simple and is depicted in Figure 3.1. There is one financial market with N assets, indexed by $j, j = 1, 2..., N$. Each assets is traded by n traders, indexed by $i, i = 1, 2, ..., n$. There are four types of traders - noise traders, mean variance traders, mean reversion traders and finally relative chartist traders. Each of these groups is characterized by a different trading strategy, which is pursued by traders belonging to these groups. The strategies will be described later.

The simulations will run in T time steps. In each time step, agents are randomly activated. When agents are activated, they perform several calculations that allow them to determine number of assets that they will try to buy/sell. They also assess the past price movements and decide on the price at which they are willing to buy/sell assets. Next, they issue limit orders to buy/sell assets (the next section will clarify all details of this process) and the market demand and supply is determined by these orders. A new equilibrium price is set on the basis of the demand and supply and orders that are not consistent with this price are cancelled. Finally, an exchange of cash and assets can be made.

The decisions of agents work in the following way. First, they evaluate their total wealth, which consists of wealth held in risky assets and the risk-free cash: $W_i =$

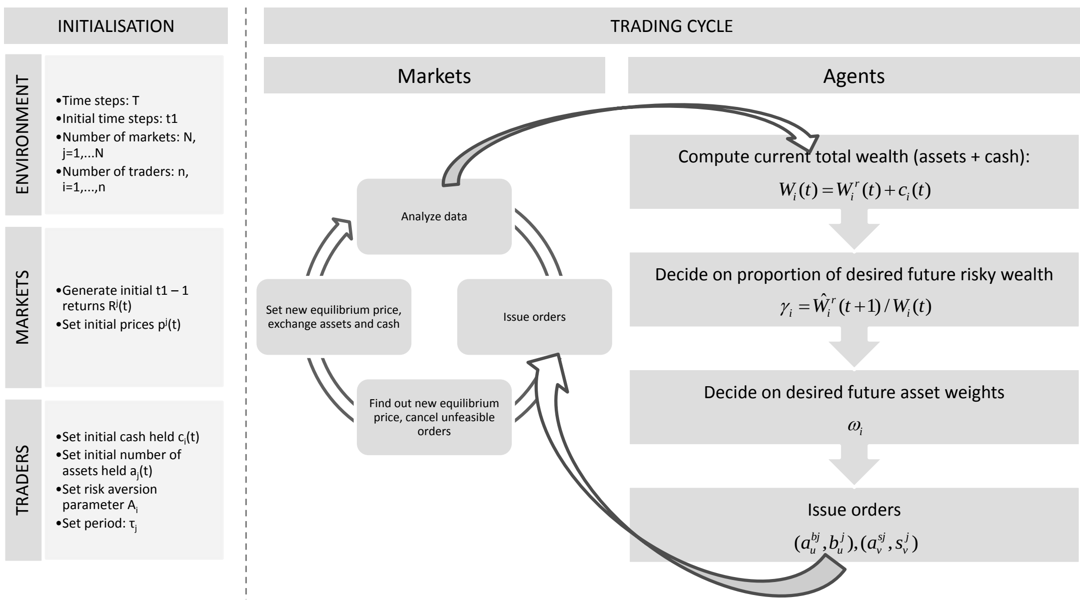


Figure 3.1: An overview of the model

$W_i^r(t) + c_i(t)$. Next, they decide on the desired proportion γ_i of risky assets held in the future on their total wealth. Usually, the more an investor risk averse is, the lower the desired proportion of risky assets on the total wealth is and vice versa. A desired risky wealth in the next time period can be determined using γ_i: $\hat{W}_i^r(t+1) = W_i(t) * \gamma_i$. In the next phase, an agent decides on the desired weights $\hat{\omega}_i^j(t+1)$ of each asset on the risky wealth. From these weights, the amount of assets that an investor wants to buy or sell can be easily determined (exact formulas will be described later on). Finally, the activated agents determine prices at which they are willing to buy or sell the assets and issue limit orders to the market.

However, this approach has a major flaw that the authors of the original model do not address. At first sight, everything written in the last paragraph might seem correct. Nonetheless, we must bear in mind that not all the trades that an investor wants to pursue materialize. For example, when an investor issues an order to sell 10 shares for EUR 10 and a new equilibrium price is EUR 9.50, this trade will not be accomplished. As a result of the chosen approach, an investor might lack some cash after a trading period and it is in contrast with an assumption of the model that leveraging (buying of assets with borrowed money) is not allowed. As an example, let us assume that an investor wants to rebalance his or her portfolio consisting of two assets, A and B. In period 1, he/she holds 10% of the risky portfolio in asset A, 90% in asset B and some amount of cash. Then assume that he or she wants to hold 90% of risky wealth in asset A and 10% of risky wealth in asset B. Therefore he or she issues an order to buy some units of A and sell some units of B. When everything goes in good way, no problem arises. However, when a new equilibrium price is bellow the limit price that he/she has chosen for asset B, the investor might lack cash because he/she did not receive money the expected money from the sale of assets B.

A solution of this problem would slow down simulations immensely so we did not do any adjustments here. But we have assessed the significance of this problem. We have run several simulations and it follows that investors usually do not owe more than 1.5 of the cash they held at the beginning of the simulation. In addition, only a small proportion of investors are affected by this problem and this problem lasts usually only to the next trading period, when they issue orders to sell almost all their shares to hold some positive cash. We do not charge these investors any interest rate in order to isolate the effects that we want to observe. Charging an interest rate would cause only some small inflow / outflow of the money in/out of the market and would only distort results of simulations.

In order to allow all the mentioned parts of the model to work, several parameters must be initiated at the beginning of simulations. Some of them have been already mentioned. Next we set returns on assets for the first $t_1 - 1$ periods, so that decision making of agents can work from the very beginning. The initial returns are set equally on each asset as a random draw from the normal distribution. Finally, we set two parameters for each agent. The first one is risk aversion parameter A_i and the second one is period τ_i. This parameter determines length of time span on which agent base their decisions.

This has been a brief overview of the model. In the following subsections, we will elaborate descriptions of various parts described in this part.

3.2 Price determination

A crucial part of every agent-based model of financial markets is price formation. Economic theory says that price is determined by supply and demand, but it proves challenging to develop a model of price formation process that would be completely realistic and also easy to implement.

Various price determination processes have been used in different agent-based models of financial markets. A survey of them is given in LeBaron (2006). We have decided to use price determination mechanism similar to that used in the Genoa market (Raberto *et al.* (2001)). This mechanism is a simple version of limit order markets, where supply and demand is matched through equilibrium price.

Each agent can issue an order either to buy or sell some quantity of assets. A limit price is associated with each order. A limit price for order to buy an asset is the highest price for which the investor is willing to buy specified amount of the asset. Conversely, a limit price for a sell order is the lowest price for which the investor is willing to sell specified amount of the asset. A new equilibrium price is price which balances supply and demand.

Formally the price determination works in the following way: at time step t, each agent $i, i = 1...n$ can issue various orders for time $t+1$. They can issue an order to sell a_{ij}^s assets j. An associated limit price is denoted by s_{ij}. Or they can issue a buy order to buy a_{ij}^b units of $j-th$ asset. Now the associated limit price is b_{ij}

In the next step, demand and supply curves are computed for each asset in order to determine new equilibrium prices $p^j(t+1), j = 1, 2, \ldots, N$ at time $t+1$. For asset j, we compute the demand curve as follows. Let U be the number of buy orders at time $t+1$ and $(a_u^{bj}, b_u^j), u = 1, 2, \ldots, U$ be a pair of buy order and the associated limit price. The market demand curve of asset j at time $t+1$ is then

$$D_{t+1}^j(p^j) = \sum_{u|b_u^j \geq p^j} a_u^{bj} \tag{3.1}$$

The market supply curve is computed similarly. Let V be the number of sell orders at time $t+1$ and $(a_v^{sj}, s_v^j), v = 1, 2, \ldots, V$ be a pair of sell order and the associated limit price. The market supply curve of asset j at time $t+1$ is

$$S_{t+1}^j(p^j) = \sum_{v|s_v^j \leq p^j} a_u^{sj} \tag{3.2}$$

The problem is that demand and supply curves formed in this way are step functions and various complications can occur. Firstly, the demand and supply curves may not cross at all. When this happens, no trade is executed and the price at time $t+1$ stays at the same level as in period t.

Next, the supply and demand curve might cross at more points, i.e. a case when more than one equilibrium price exists might arise. The authors of the Genoa market handle this problem by setting the new price at the level of arithmetic average of different equilibrium prices. For the sake of simplicity of implementation of our model and the practical time demands of the simulation, we have decided to set the new price at the level of the lowest equilibrium price. We think that this does not cause any major bias to the results of our model, since this case occurs only seldom and equilibrium prices range over a short interval.

Finally, the demand for an assets only rarely equals the supply, given the new equilibrium price. Let $D^* = D(p^*)$ be the demand for an asset at the new equilibrium price and $S^* = S(p^*)$ be the supply of an asset at the new equilibrium price. We make the following assumption: If $D^* > S^*$, S^* assets are traded. Orders to buy the remaining $D^* - S^*$ units of shares are randomly cancelled. On the other hand, if $S^* > D^*$, D^* assets are exchanged and orders to sell the remaining $S^* - D^*$ units of shares are randomly cancelled again.

Now the question is how limit orders are determined. The amount of assets that each agent wants to buy is given by the amount of assets that they currently hold, by their cash wealth and most importantly by trading strategy that they employ. We will return to this problem in later sections. On the contrary, the way that limit prices are set is the same for all agents. Let $p(t)$ be the last observed price. Then we set the buy order limit price of agent i as

$$b_i^j = p(t)N_i(\mu, \sigma_i), \tag{3.3}$$

where $N_i(\mu, \sigma_i)$ is a random draw from the normal distribution with mean $\mu = 1.01$ and standard deviation σ_i. The standard deviation is proportional to historical volatility:

$$\sigma_i = k\sigma(T_i),$$

where k is a parameter and $\sigma(T_i)$ is the standard deviation of log-returns[1] in the last T_i periods. In our simulations, we have chosen T_i constantly for all traders and it was set as 20. Simulations worked best for parameter k set as 3.5.

Similarly, the sell limit price is given by

$$s_i^j = \frac{p(t)}{N_i(\mu, \sigma_i)} \tag{3.4}$$

[1]log-return in period t is defined as $r_i = \log(p(t)/p(t-1))$

Setting the limit prices in this way has various convenient properties. Incorporating the price volatility is useful in that in times of high price volatility, agents tend to ask / demand wider range of prices. Next, due to the setting prices by these formulas, buy limit prices tend to be higher than the current price and sell limit prices tend to be lower than the current price. As a result, in the case of higher demand, prices are rising and falling in the case of higher supply. Next, a kind of bid-ask spread is introduced. However, we do not consider transaction costs, so this spread is further not taken into account.

Before we progress to describe the decision making of each type of investors, we are going to provide a few examples to illustrate how the price determination works in our model.

3.2.1 Illustrations of price determination

In this section we illustrate various features of the price determination in our model. First, we present a concrete example of how the price can be determined and next we show the results of new prices given a constant initial set-up.

As we have already stated, each trader can issue an order to buy or sell some units of assets. An example of limit orders of 20 agents is presented in table 3.1. A minus sign before an order marks a sell order, otherwise a buy order is issued. For example, the first trader wants to sell 3 units of the asset for price higher or equal to 10.61. The trader number 3 wants to buy 4 units of the assets for price lower or equal to 9.59.

Trader	Limit order	Limit price	Trader	Limit order	Limit price
1	-3	10.61	11	-5	9.79
2	-4	9.81	12	-3	10.04
3	4	9.59	13	5	10.52
4	-4	9.85	14	-5	9.91
5	1	10.42	15	-4	10.28
6	5	9.77	16	1	9.91
7	-3	9.98	17	-1	9.61
8	-2	10.88	18	4	9.74
9	1	9.82	19	-3	10.36
10	3	10.19	20	1	9.88

Table 3.1: Illustration of limit orders

A market demand and supply can be derived from the individual orders. This is done in table 3.2 accoring to equations 3.1 and 3.2. We must bear in mind that the market demand and supply functions are not defined only in these discrete points. The domain of the functions are all (positive) prices. The demand function is continuous in the points listed from the left and the supply function is continuous in the points listed from the right. However, we make an assumption that the tick size is 0.01 as is common in real stock exchanges. It means that the smallest change in share prices that can be recorded is 0.01 units of currency.

p	9.61	9.79	9.81	9.85	9.91	9.98	10.04	10.28	10.36	10.61	10.88
S(p)	1	6	10	14	19	22	25	29	32	35	37
p	9.59	9.74	9.77	9.82	9.88	9.91	10.19	10.42	10.52		
D(p)	25	21	17	12	11	10	9	6	5		

Table 3.2: Market demand and supply

The market supply and demand functions from this example are depicted in Figure 3.2. The equilibrium price (i.e. price for which the absolute value of difference between demand and supply is minimal) is 9.82. At this price, 10 units of shares are supplied and 12 are demanded. As stated above, since there is an excess demand in the market, 10 units of share will be traded and limit orders for two units of shares will be randomly cancelled. A new equilibrium price will be set at 9.82.

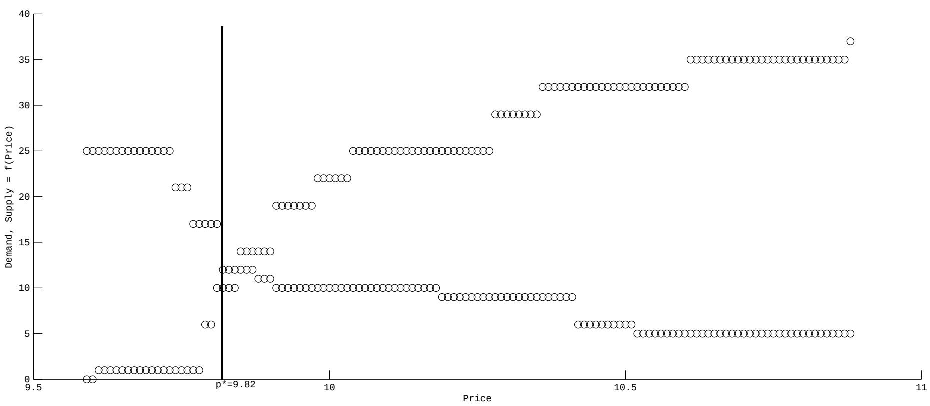
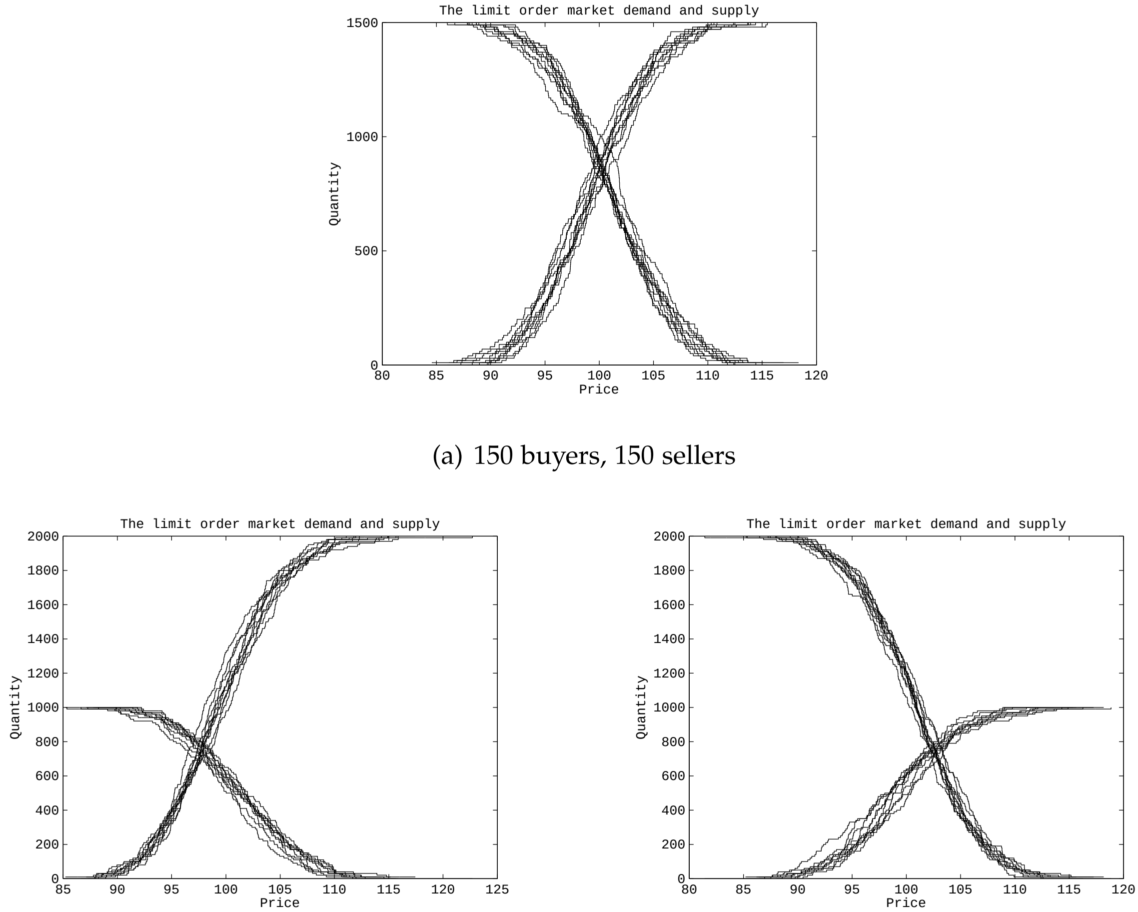


Figure 3.2: An illustration of limit orders

Since the traders set the limit prices according to 3.3 and 3.4, the resulting clearing price can vary even though the limit orders are the same. This point is illustrated in



(a) 150 buyers, 150 sellers

(b) 200 buyers, 100 sellers

(c) 100 buyers, 200 sellers

Figure 3.3: Various cases of demand and supply in presence of 300 traders and 10 time steps

figure 3.3 and table 3.3. In both cases, we have run a simple simulation, when each trader sells or buys 10 units of assets.

In order for the figure to be legible, simulations where only 300 traders are presented have been run 10 times. The limit prices are set by the algorithm described above, with the initial price of 100 and the standard deviation of returns of 0.05. In the first picture, 150 buyers and sellers issue limit orders to buy or sell 10 units of an asset. In the second picture, the numbers change to 200 and 100 respectively and finally in the third picture, the simulation was run with 100 buyers and 200 sellers. The movement of the clearing price is the same as we expected, although it is not the same in each case.

To make the dispersion of resulting equilibrium prices more palpable, we have run a set of 20 simulations, whose results are displayed in table 3.3. Again, each trader

Std./buyers, sellers		1500, 1500	1000, 2000	1250, 1750	1750, 1250	2000, 1000
0.01	mean(eqp)	99.9995	98.9776	99.4421	100.559	101.0318
	std. dev(eqp)	0.2778	0.2615	0.2772	0.2748	0.2642
0.05	mean(eqp)	99.9949	97.5155	98.7662	101.2461	102.5443
	std. dev(eqp)	1.1563	1.1474	1.157	1.181	1.2004
0.1	mean(eqp)	99.9883	95.487	97.7412	102.3067	104.712
	std. dev(eqp)	2.3393	2.242	2.229	2.3716	2.4447
0.2	mean(eqp)	99.9994	91.5651	95.7317	104.4759	109.1774
	std. dev(eqp)	4.5344	4.1475	4.4337	4.8978	4.9963

Table 3.3: Properties of price determination mechanism

wants to buy or sell 10 units of the asset and the initial price was set as 100. The total number of traders was changed to 3,000 and the number of cycles to 10,000. In the header of the table, the numbers of buyers and sellers are written, respectively. In the first column, the standard deviations that enter the formulas 3.3 and 3.4 are stated. We can see that the new equilibrium price almost remains at the same level as the aggregate number of demanded asset equals the number of supplied asset. In addition, this equilibrium price tends to be more widely dispersed as the standard deviation of returns on the asset increases.

Similarly, in the presence of excess supply (columns 3 and 4), the equilibrium price tends to decline as we would expect. The extent to which it declines is positively related to the standard deviation of returns on the assets. The same holds for the dispersion of the equilibrium price.

As far as the direction of the equilibrium price movement in the presence of excess demand (columns 5 and 6) is concerned, the opposite holds than for the excess supply - the equilibrium price tends to rise. Again, the price tends to be more dispersed as the standard deviation of returns on the asset increases.

3.3 Typology of Traders

There are four types of traders in our model. Traders of each type pursue a different trading strategy. Noise traders trade "on noise", that is on the information that they deem as correct, although it might not be so. Mean variance traders change their portfolio in order to maximize their utility. Mean reversion traders believe that the returns on stocks are on average constant and behave in order to exploit deviations from this mean value. Finally, relative chartists invest in the most profitable assets. In the first sets of simulations, behaviour of traders is constant with respect to the amount of cash they invest in shares. Next we add some elements that will change amount of cash allocated in risky assets according to attitude of traders towards risk.

3.3.1 Noise traders

> *"Noise makes financial markets possible, but also makes them imperfect."* (Black, 1986)

Noise traders form an important part of our model. Their behaviour is characterized by random adjustment of assets weights in portfolio and trading in order to attain portfolio corresponding with these weights.

The philosophy behind noise trading was given by F. Black in his widely-respected paper Noise[2](Black (1986)). He asserts that noise (everything in contrast with true information) makes the financial markets liquid and enable trading with individual securities. If there were no noise, agents owning some piece of information would not be willing to enter trade, since the opposite part of the trade also possesses another piece of information and this might challenge veracity of the owned information. In order to prevent losses, traders would not trade at all. Noise makes also information more valuable and the traders are willing to invest in information in order to discern noise from true information. Finally, noise makes trade profitable, since informed traders usually trade with noise traders (who think that noise on which they trade is the true information) and these lose in the long-run.

We model noise traders as zero-intelligence traders in Cincotti *et al.* (2005). As any other traders, also noise traders are initially endowed with cash and assets. At time t, the i agent holds $c_i(t)$ units of cash and $a_i^j(t)$ units of $j-th$ asset. The price

[2] Black deals with more issues than only financial markets in this paper. Topics as inflation, international trade and business cycles are also addressed.

of asset j at time t is $p^j(t)$. Therefore the risky part of the $i-th$ trader is worth $W_i^r(t) = \sum_{j=1}^{N} a^j(t)p^j(t)$. The total wealth is the sum of risky wealth and cash held by the investor: $W_i(t) = c_i(t) + W_i^r(t)$.

The weight $\omega_i^j(t)$ of asset j on the risky wealth of the $i-th$ investor at time t is expressed as $\omega_i^j(t) = p^j(t)a_i^j/W_i^r$. We assume that $\omega_i^j(t) \geq 0 \forall j \in \{1, 2, \ldots, N\}$ (short selling is not allowed) and $\sum_{j=1}^{N} \omega_i^j = 1$ (leveraging is also not enabled).

At time t, noise trader i is activated with probability π. First, desired wealth held in risky assets at time $t+1$ is determined as a random fraction of the total wealth at time t: $\hat{W}_i^r(t+1) = \gamma_i W_i(t)$, where γ_i is a random draw from the uniform distribution $U(0,1)$. This amount is only desired, or a target and may not be met in the future, depending on the amount of assets being traded and the future prices.

The desired weights of each asset $\hat{\omega}_i^j(t+1)$ in the risky portfolio at time $t+1$ are again determined randomly, subject to the above mentioned constraints on short-selling and leveraging. The desired amount of the asset j held by trader i at time $t+1$ is then:

$$\hat{a}_i^j(t+1) = \left\lfloor \hat{\omega}_i^j(t+1) \frac{\hat{W}_i^r(t+1)}{p^j(r)} \right\rfloor, \tag{3.5}$$

where $\lfloor \ldots \rfloor$ means the integer part of the expression.

The difference between the desired and currently held amount of the asset j is

$$\Delta_i^j(t+1) = \hat{a}_i^j(t+1) - a_i^j(t).$$

If the $\Delta_i^j(t+1) > 0$, the trader i issues a buy order to buy $\Delta_i^j(t+1)$ units of the asset j. Conversely, should the $\Delta_i^j(t+1) < 0$, a limit order to sell $\Delta_i^j(t+1)$ units of $j-th$ asset is issued. The corresponding limit price is determined in the way described in Section 3.2.

3.3.2 Mean variance traders

The behaviour of mean variance traders is characterized by adjusting their portfolio in order to maximize their utility.

Let R_j be a random variable of return on the asset j. We will denote the expected return on the j-th asset as μ_j and $\mu = (\mu_1, \ldots, \mu_N)^T$ the vector of expected returns. Let $\Sigma = cov(R_i, R_j), i, j \in \{1, \ldots, N\}$ is a covariance matrix of returns on the assets. We can then write the expected return on portfolio as $E(R_p) = \mu^T \omega$, where ω is a vector of proportion of assets in the portfolio. The portfolio variance is $Var(R_p) = \omega \Sigma \omega^T$.

Mean variance traders choose the vector of weights ω so as to maximize utility. We assume that their utility is characterized by the CARA utility function in the form of:

$$u(W) = 1 - e^{-\phi W}, \tag{3.6}$$

where W is wealth of an investor and ϕ is his or her risk aversion parameter.

Lemma 1. *Let $U(W)$ be a CARA utility function of the form 3.6. Let W be normally distributed with the mean μ_W and variance σ^2. Then the function $E[U(W)]$ is an increasing monotonic transformation of $u(E[W], \sigma^2) = \mu_W - \frac{\phi}{2}\sigma^2$ and their maximization is equivalent.*

Proof of the lemma. We know from microeconomic theory that the results of optimization of a utility function are equivalent with optimization of an increasing monotonic transformation of the utility function. It is therefore sufficient that U is a monotonic transformation of u.

As the wealth is normally distributed, its probability density function is

$$f(W) = \frac{1}{\sqrt{2\pi}} exp(-\frac{(W-\mu_W)^2}{2\sigma^2}).$$

Then the expected utility of wealth may be written as

$$E[U(W)] = \int_{-\infty}^{\infty} U(W) f(W) dW = \int_{-\infty}^{\infty} -\frac{1}{\sqrt{2\pi}} exp\left[-\phi W - \frac{(W-\mu_W)^2}{2\sigma^2}\right] dW$$

The argument of the exponential function on the right can be rewritten as

$$-\frac{2\sigma^2\phi W + W^2 + \mu_W^2 - 2W\mu_W}{2\sigma^2} = -\frac{\left[W - (\mu_W - \phi\sigma^2)\right]^2 + 2\phi\mu_W\sigma^2 - \phi^2(\sigma^2)^2}{2\sigma^2}.$$

Each term in the last expression, except of W, is deterministic. We can therefore express $E[U(W)]$ as

$$E[U(W)] = -exp(\phi\mu_W - \frac{\phi^2}{2}\sigma^2) \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} exp\left(\frac{\left[W - (\mu - \phi\sigma^2)\right]^2}{2\sigma^2}\right) dW.$$

The value of the integral is one, since it is the value of cumulative distribution function of normally distributed random variable with mean $\mu - \phi\sigma^2$ and variance σ^2 in infinity. Finally, we can think of the expected utility function as an increasing monotonic transformation of the function in the assertion of this lemma:

$$E[U(W)] = -exp(\phi\mu - \frac{\phi^2}{2}\sigma^2) = u(\mu - \frac{\phi}{2}\sigma^2).$$

$\square$

According to the previous lemma, mean variance traders are maximizing this function:

$$u(\omega) = \omega_i^T \mu - \phi \omega_i^T \Sigma \omega_i. \tag{3.7}$$

The desired weights of the assets in time $t+1$ are therefore given by the solution of the maximization problem:

$$\hat{\omega} = \arg\max_{\omega} u(\omega) = \omega_i^T \mu - \phi \omega_i^T \Sigma \omega_i,$$

$$s.t. \omega_j \in R_{+,0}, \sum_{j=1}^{N} \omega_j = 1.$$

Again, we do not permit short selling or leveraging. Practically the decision of mean variance traders proceeds in the following way. First, the total risky wealth W_i^r and the total wealth W_i is computed for the $i-th$ mean variance trader at time t in the same way as for the noise traders. Next, the desired weight of the risky wealth is determined as a fraction of the current total wealth: $\hat{W}_i^r = \gamma_i W_i(t)$. As the next step, the weights of risky assets $\hat{\omega}$ are computed by 3.7 and the desired number of assets held in the next period are given by 3.5. Finally, the limit orders are issued according to whether Δ is higher or lower than zero.

Several modifications of this basic algorithm are made in order to make mean variance heterogeneous and the model agent-based. First, the proportion of the desired risky wealth on the total wealth is a function of the risk-aversion parameter: $\gamma_i = 0.6 + 0.3/\phi_i$ Cincotti *et al.* (2003), where $\phi \sim U(1,2)$. This means that the more an investor is risk-averse, the less wealth will be held in risky assets. Next, the agents are activated only with some probability π. When activated, they use historical returns in the last τ_i periods to estimate expected returns and covariance matrices of returns.

3.3.3 Mean reversion traders

Mean reversion traders believe that asset returns follow the Ornstein-Uhlenbeck process, which is the simplest case of a mean reverting process. Ornstein-Uhlenbeck process is a stochastic process described by

$$dx = \eta(\bar{x} - x)dt + \sigma dz,$$

where η is the speed of reversion, $\bar{x}$ is the mean level to which x tends to revert and σ is the standard deviation of x. The size of change in x is proportional to its distance from the mean and the speed on the η parameter (Dixit *et al.* (1994)).

The mean reversion traders expect that when the current price is above the mean price, it will return to the mean level in the future. Conversely, when the current price is lower than the mean price, it is expected to grow to the mean price. These two states are linked to the negative and positive return, respectively.

In our model, each mean reversion trader is activated only with some probability π and their trading strategy is following. Firstly, they compute the vector of distance of actual prices from their means in the last τ_i periods: $d = \bar{p^j} - p^j(t)$. Each element of vector d has the same sign as the expected return. Next, the investors compute the covariance matrix Σ of the asset prices during the last τ_i periods. Finally, the desired portfolio weights are given by

$$\arg f(\hat{\omega}) = \hat{\omega}_i^T d - \phi_i \hat{\omega}_i^T \Sigma \hat{\omega}_i$$

3.3.4 Relative chartists

Relative chartists adjust the weights of assets in their portfolios so that the most profitable assets have the largest weight. Let R^k be return on the $k-th$ asset during the last τ_i time steps. Let R_i be the return on the i-th trader risky portfolio during the same period. The desired weight of the $k-th$ asset in the $i-th$ investor's portfolio is set as in Cincotti *et al.* (2003, p. 230):

$$\hat{\omega}_i^k(t+1) = \omega_i^k(t)\left[\eta_i\left(\frac{R^k}{R_i} - 1\right) + 1\right], \tag{3.8}$$

where $\eta_i \sim U(0.1, 0.15)$ is a learning parameter.

It is worth showing that the desired weights sum to one:

$$\sum_{k=1}^{N} \hat{\omega}_i^k(t+1) = \sum_{k=1}^{N} \omega_i^k(t)\left[\eta_i\left(\frac{R^k}{R_i} - 1\right) + 1\right] = \tag{3.9}$$

$$= \eta_i \left[\underbrace{\sum_{k=1}^{N} \omega_i^k(t)\frac{R^K}{R_i}}_{=\frac{R_i}{R_i}=1} - \underbrace{\sum_{k=1}^{N} \omega_i^k(t)}_{=1}\right] + \underbrace{\sum_{k=1}^{N} \omega_i^k(t)}_{=1} = 1 \tag{3.10}$$

Unfortunately, the behaviour of relative chartists cannot be based solely on the equation 3.8. This is reasonable only in a special case when returns on all assets are positive and the return on the portfolio is therefore positive as well. The authors of the Genoa market do not mention the behaviour of relative chartists in other cases, so we have elaborated it a bit deeper.

We discern four cases that can occur and the behaviour of relative chartists is contingent on these states:

1. $R_i > 0 \wedge \forall k : R^k > 0$: this is the simplest case, when both returns on all assets and on the portfolio was positive during the last τ_i periods. The equation 3.8 can be applied and the largest weight on the best performing asset results.

2. $R_i > 0 \wedge \exists l : R^l < 0$: one or more assets had negative returns during the last τ_i periods, but the return on the whole portfolio was positive during that period. We proceed in four steps:

   (a) We still apply equation 3.8 to get $\hat{\omega}_1^k$ but sometimes it can happen that the desired weight $\hat{\omega}_1^l$ is negative. If this happens, we proceed further.

   (b) Since short selling is forbidden, we set $\hat{\omega}_2^k = \max\left\{0, \hat{\omega}_1^k\right\}$. Let us further define sets of indices where returns were positive or negative, respectively, in the last τ_i periods: $\Omega_1 = \left\{\ell = 1, ..., N : R^\ell > 0\right\}$, $\Omega_2 = \left\{\ell = 1, ..., N : R^\ell < 0\right\}$.

   (c) Because the weights $\hat{\omega}_2^k$ do not sum to one now, we adjust them in the following way: $\hat{\omega}_3^k = \frac{\hat{\omega}_2^k}{\sum_{m=1}^N \hat{\omega}_2^m}$

   (d) Next, we assume that relative chartists do not want to change their portfolio very dramatically (and they are not willing to sell all of some type of assets, for the sake of diversification, transaction costs etc.). Therefore we scale the weights further as a convex combination of ω^k and $\hat{\omega}_2^k$: $\hat{\omega}_3^k = \omega^k + 0.5 * \hat{\omega}_2^k$. It means that we obtain weights $\hat{\omega}_3^k$ that lie between ω^k and $\hat{\omega}_2^k$ - between the old weights and weights that might be dramatically different from the former ones.

   (e) Finally, we need to take into account that some assets with negative returns have worse returns than other assets with negative returns. We therefore adjust weights 3 for assets with negative returns as:

$$\hat{\omega}^m = \frac{\sum_{\ell:\ell\in\Omega_2} \hat{\omega}_3^\ell}{weight}, m \in \Omega_2,$$

   where weight is weight proportional to the extent to which return is negative:

$$weight = \frac{\frac{1}{-R^k}}{\sum_{\ell:\ell\in\Omega_2} \frac{1}{-R^\ell}}.$$

Nothing more changes for assets, whose returns were positive: $\hat{\omega}^m = \hat{\omega}_3^m$

3. $R_i < 0 \wedge \exists l : R^l > 0$: the return on portfolio was negative but some assets with positive returns exist. The desired weights given by formula 3.8 are exactly opposite from those that we would expect: they are positive for assets providing negative returns and negative for positive returns. In addition, the worst performing assets are given the largest weight. We adjust this flaw in four steps:

   (a) Apply formula 3.8 and obtain weights $\hat{\omega}_0^k$.

   (b) Since $\hat{\omega}_0^k$ is negative for assets that had positive returns, we set $\hat{\omega}_1^k = -\hat{\omega}_0^k$.

   (c) Proceed as in 2. (b) - (e).

4. $R_i < 0 \wedge \forall l : R^l < 0$: returns on the portfolio and all assets were negative at the same time. The formula 3.8 can be applied with one modification. Since the largest desired weight is given on the worst performing assets, we put inverted values of returns on assets into the formula:

$$\hat{\omega}_i^k(t+1) = \omega_i^k(t) \left[\eta_i \left(\frac{R^k}{\frac{1}{R_i}} - 1 \right) + 1 \right],$$

3.4 IES Market - market with endogenous risk aversion

The first sets of simulations that we have run are only slight adjustments of the original Genoa market, although we have focused on more properties and modifications than the authors of the market. In the next step, we modify behaviour of three groups of traders. The behaviour of noise traders will stay unchanged, since they create a "thermal bath" that allows the market to be liquid. We introduce endogenous risk aversion and cash outflow that follows from it. Both ideas are inspired by Feldman (2008). A working name of this final model is 'IES Market'. Nevertheless, we are deeply indebted to the authors of the Genoa market, who invented the core ideas of the model.

The first modification that we make is the way that mean variance and mean reversion traders estimate volatility and covariances of asset returns. Instead of formulas known from elementary courses of statistics, the traders will now use more sophisticated EWMA (exponentially weighted moving average) model. This method has two advantages over the standard volatility estimation. First, the volatility reacts faster to shocks since the most recent observation have the highest weight in the covariance/variance estimate. Next, if the shock does not persist, it dies out quickly since the weights on observations decline exponentially.

The formula for EWMA variance calculation is following:

$$\sigma = (1-\lambda)\sum_{t=1}^{T}\lambda^{t-1}(r_t - \bar{r})^2$$

Similarly, EWMA covariance is computed as:

$$\sigma_{1,2}^2 = (1-\lambda)\sum_{t=1}^{T}\lambda^{t-1}(r_{1,t} - \bar{r_1})(r_{2,t} - \bar{r_2})$$

The parameter $\lambda \in (0,1)$ is often called the decay factor and we have set it as 0.94, as in Morgan and Reuters (1995, p.80). By the use of EWMA to forecast variances and covariances of returns, the forecasts are more accurate and flexible.

Next we wish to incorporate cash inflow in the market and outflow out of it. The decisions of agents on the level of cash that they want to invest in risky assets are based on riskiness of the market. In times of bull market, they usually invest more because they expect asset prices to rise further and thus a profit is possible. On the other hand, in times of bear market, some agents want to prevent large losses from holding their assets and they thus sell some assets they hold. But some traders do not sell because they believe that the markets will rally and it would make more harm to them to sell at the moment.

It is well known from behavioural finance (Lo (2004) gives some exposition and references on behavioural finance) that people are usually loss averse. A decrease in utility caused by a loss of $EUR10$ is higher than an increase in utility when $EUR10$ is found. That is why we introduce a loss function in the model. This function captures only losses that investors incur. We define an individual investor's loss function as: $\ell_i^t = \min\left\{R_t^{p_i}, 0\right\}$, where $R_t^{p_i}$ is return on the i-th investor's portfolio during period t.

We aggregate individual investors' loss functions as simple mean and obtain $\bar{\ell}(t)$. The loss function of the market is given as an exponential average of loss functions in last periods: $\hat{L}(t) = \lambda\bar{\ell}(t) + (1-\lambda)\hat{L}(t-1)$. Again, we have set λ as 0.94. Exponential averaging reflects that most recent losses are still remembered but memories on those in the past die out very quickly.

Investors respond to negative mood in financial markets differently. It depends on information, believes and nature of an investor whether he or she will sell all their assets when first negative news arrive. We therefore introduce a perception index to market losses $\beta_i \in < 0, 1 >$. We assume that even if $\beta_i = 0$, investors still react to market losses. When $\beta_i = 1$, an investor sells or buys assets very frequently as a result of change in performance of all the investors. Nevertheless, we make an assumption that investors want to hold at least 0.1 and no more than 0.9 of their wealth in risky assets.

Next, we distinguish between two scenarios:

3.4.1 Approach 1 - aversion to absolute levels of the loss function

During a series of simulations, we have found that the loss function of the market $\hat{L}(t)$ rarely exceeds value of 0.02. This number might seem very low but we must bear in mind that this loss is incurred during mere one period. We therefore adjust further steps to this number. We assume that traders allocate their wealth to risky wealth according to the following equation:

$$\hat{\gamma_i(t)} = 0.1 + \frac{1.6}{1 + exp(-\hat{L}(t) * (50 + 100 * \beta_i))}$$

The function for given propensity parameters of 0, 0.5 and 1 respectively is plotted in Figure 3.4. As expected, for a given value of the loss function, investors with higher propensity want to invest less cash than investors with lower propensity.

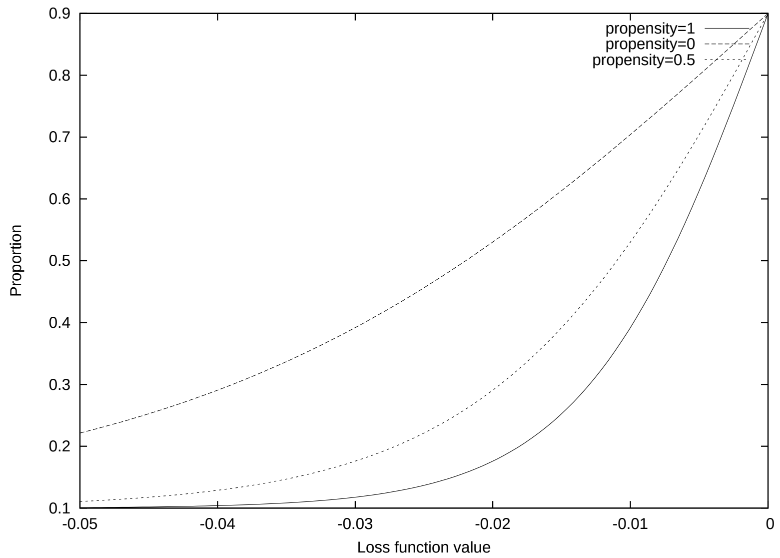


Figure 3.4: Proportion of cash invested in risky assets for three propensity parameters

3.4.2 Approach 2 - aversion to changes in levels of the loss function

In the first approach, agents changed their behaviour according to the absolute level of the loss function. They could therefore change proportion of desired risky wealth on their total wealth very quickly. This is probably not really realistic. We do not expect somebody to invest 80% of their wealth in shares one day, 15% on the second and 60% on the third day. We have therefore created another way of traders' behaviour.

In this approach, agents respond to changes in the loss function, not to its absolute values. We define change in the loss function as $\Delta_l = lossF(t) - lossF(t-1)$. When $\Delta_l > 0$, losses among investors have diminished during the last period and some of them are expected to invest more cash in shares. On the other hand, when $\Delta_l < 0$, losses have deteriorated and some investors will sell some of their shares to decrease proportion of their risky wealth.

The traders react to changes in the loss function differently. In order for them to react at all, the absolute value of change must be higher than a specified threshold level. We therefore split up the traders into three groups according to their propensity: agent with propensity $\beta_i \in< 0; 1/3)$ are categorized into the first group, agents with propensity $\beta_i \in< 1/3; 2/3)$ belong to the second and finally agents with $\beta_i \in< 2/3; 1 >$ belong to the third group. With each group, a threshold level $(T_g, g \in 1, 2, 3)$ is associated. Finally, traders change their behaviour only if absolute value of Δ_l exceeds the threshold associated with their group.

Another ingredient that we add to the model is a good news indicator S_t. The indicator will represent the mood of the news and economic indicators. It can take on 3 values: $S_t = -1, 0, 1$ representing bad news, neutral news and good news respectively. When traders suffer losses and they believe that macroeconomic fundamentals are good, they will respond less than if common consensus among media, statistical indicators etc. was that an economy is heading to a recession. The good news indicator will change randomly every 100 periods and will have influence on decisions of traders only when its value is different from zero.

All these elements are combined in the following way. If $|\Delta_l| > T_g$, where g indicates the group to which agent i belongs, the i-th agent will change the desired proportion of risky wealth:

$$\Delta\hat{\gamma}_i = \Delta_l * \beta_i * c_1 + S_t * \max\{0, S_t * \operatorname{sgn}\Delta_l\} * c_1 * \beta_i,$$

where c_1 and c_2 are constants. The second term in the equation indicates that in times of good news from the real economy (S_t=1) and financial markets ($\Delta_l > 0$), agents will increase proportions of their risky wealth more than if they heard only good news from the stock markets. In addition, good news from the real economy do not change traders' decisions in times of increased losses in the stock market. On the other hand, when the news are bad from both real and financial economy, traders will respond more than in the just mentioned case. In times of neutral news, agents respond only according to the first term in the equation.

We have made one more small adjustment. Parameters c_1 and c_2 higher when $\Delta_l < 0$ than when $\Delta_l > 0$. This is consistent with the prospect theory (by Kahneman and Tversky, references are provided in Lo (2004)).

Chapter 4

Simulations

This chapter presents results of simulations of the model for different set-ups. In the first simulation, only the simplest of the traders, noise traders will be present. In the next 3 simulations, the remaining sorts of traders will be added to the noise traders but only one sort at once, so two kinds of traders will be present in these simulations. The first part of the simulations will conclude by putting all categories of traders into the market. In the next step, we will make the modifications of behaviour of traders and present another two sets of simulations (IES Market 1, IES Market 2).

For each set of simulations, we have run 10 simulations with 10300 time steps (the first 300 were trimmed and thrown away). Agents invested in 4 different assets, so finally we obtained 40 time series of asset returns. These time series were further tested and the results of the tests are in the Appendix.

4.1 Simulation 1 - Noise market

In the first set of simulations, only noise traders are present in the market. The proportion of desired risky wealth on their total wealth is random, as well as desired weights of each asset in their portfolio. A typical price path as well as other characteristics of the market can be seen in the Appendix on page 46.

Distributions of returns are obviously characterized by fat tails (kurtosis is higher than 3 for each asset in the market in every simulation). In addition, the Jarque Bera test rejected normality of asset returns. This test also rejected hypothesis of normality of returns during shorter time spans, say 500 periods, in most cases. This is a bit different in simulations that follow. Volatility clustering is also present, which is supported by large positive differences between all ARCH test statistics and the critical value of the test. This fact is also backed by strongly positive autocorrelations of absolute returns. These absolute returns are significant at long lags, which supports the hypothesis that share returns are a long memory process. We have tested it only heuristically by finding the first lag at which autocorrelations of absolute returns are insignificant. The results are also provided in the Appendix.

Autocorrelations of raw returns are significant at various lags. This comes as no surprise because of the characteristics of the market. On average, the amount of cash invested in assets is constant, so share prices cannot rise infinitely. In fact, they must return back and this mean reverting behaviour must be in some way captured by significant autocorrelations.

It is interesting to see what changes as a result of a different market set-up. The first feature that we change is proportion of wealth that traders invest in risky assets. So far, this parameter changed according to the uniform distribution with parameters (0,1). Now, we diminish this range so that they invest between 0.3 and 0.7. Because the amount of money invested in assets now does not change so abruptly, the asset prices do not change very dramatically as well. Therefore kurtosis does not rise very faraway above 3. Long memory is also affected, the first lag at which autocorrelation is insignificant shifts usually below 50.

Now we return the range of proportion back to (0,1) interval and increase the number of traders in the market (or equivalently, probability of their activation). As the number of traders in the market rises, the average amount of cash invested in shares becomes more and more stable, according to the law of large numbers. The law of large numbers has another effect, that desired weights of held assets stabilizes as well. Distributions of returns therefore approaches normal distribution if we look at returns during shorter time spans, e.g. 1,000 periods. Normality was rejected only in small number of cases. However, if we look at longer periods, normality is rejected again, because of the volatility clustering (variance of returns does not stay constant and hence returns do not come from the same distribution). Another distinction when more traders are added to the market is that long memory of absolute returns becomes "shorter". Autocorrelations of absolute returns become insignificant usually after 8 lags.

4.2 Simulation 2 - two types of traders

In this set of simulations, we have set the market in three ways. In each simulation, there were 1,000 noise traders and 1,000 of the other type of traders. The simulations cannot be run in presence of the sole type of traders. For example, if we ran a simulation with mean-variance traders only, they would want to buy the most profitable share. If they did it at one period, they might have found a counterparty that would be willing to sell them some of this share. However, only small amount of traders would be willing to do this. As a result, price of this share would rise very steeply. In the next period, even more traders would want to buy this asset and this would continue until everybody would want to buy this share (and it would be in only a few periods). The same holds for less profitable shares. Therefore there would not be any driving force that would make share prices volatile. That is why noise traders are needed in each simulation of the Genoa market and it is completely consistent with the ideas in Black (1986).

The first striking fact of the results of Simulation 2.1 (where mean-variance investors trade along with noise traders) is the path of share prices. It is quite smooth and remotely resemble to the sine wave. This is caused again by the fact that share prices are mean reverting (which is caused by cash constraint) and by the trading strategy of mean-variance traders. When a share is profitable (returns are positive), most of them will buy it and cause it to be even more profitable. However, the returns will diminish at some point and the traders will stop buying this share so heavily and shift their preferences towards another share. The price of the former one will finally start to decline. This process has another result - very significant positive autocorrelations of orders up to 10 or more lags. It might be also a cause of "very long memory" of absolute asset returns. None of the 40 time series of returns did not have autocorrelation of absolute returns insignificant at leg smaller than 1545. As far as other characteristics are concerned, there is no significant difference between this and the first simulation - distributions of returns are still leptokurtic and ARCH effect is found as well.

The paths of asset prices in Simulation 2.2 (where mean-reverting investors trade with noise traders) is a bit different but bears features that could be expected from the known behaviour of of mean-reverting traders. As mean-reverting traders sell assets, whose prices have gone up and buy assets whose prices have gone down in the last period, their behaviour causes the prices go in opposite direction from that in the previous period. It is then not surprising that autocorrelation is significantly positive at lag 1, significantly negative at lag 2 etc. It becomes insignificant usually after four lags. The shapes of share price paths reflect this feature. As stock prices are almost perfectly mean reverting and "controlled" by mean-reverting traders from large deviations, fat tails are almost not present. In fact, the distribution of only one of 40 share returns was platykurtic. Nevertheless, the Jarque Bera test rejected normality in all 40 cases. But as in the case of the adjusted Simulation 1, when we take shorter time spans (such as 1,000 periods), normality is usually not rejected.

Finally, Simulation 2.3 (market with relative chartists and noise traders) does not have such distinctive features as the previous cases. There might be so many cases of price behaviour, that behaviour of relative chartists might resemble the behaviour of noise traders (and the share price paths are also most similar to the Simulation 1). To give an example, let us say that one relative chartists trades with 60 time step long period and one another trader with 30 time steps long period. Let us further assume that prices 60 and 30 periods ago and the current price are in this relation $p_{t-60} < p_{t-30} > p_t$. It means that the first trader might deem ownership of this share profitable and the second one unprofitable at this moment and the traders' behaviour will be in the opposite directions. This was only one example of what might happen. If we realize that similar antagonistic conditions might arise at one moment, it seems that behaviour of relative chartists is random on aggregate. This consideration can explain similar patterns of autocorrelation functions in this and the first simulation. One characteristic is however different. Kurtosis is lower in this case and it is probably caused by higher amount of active traders in Simulation 2.3 and the fact that cash in the stock market is more stable in this case.

4.3 Simulation 3 - four types of traders

It is very difficult to find a mix of parameters that would make Simulation 3 (where all the mentioned types of traders trade with each other) realistic. If we set the market as in Cincotti *et al.* (2003) (10,000 noise traders, 10 mean variance traders, 10 relative chartists, and 10 mean-reversion traders), the results are almost identical as in Simulation 1 for obvious reasons. However, we were curious about what would happen if more traders of the three latter mentioned types were added.

We have set the market in a way that 1,500 noise traders trade with 300 mean-variance, 300 mean-reversion and 300 relative chartist traders. The results are given again in the Appendix. The first noticeable feature of this set-up is that the price path resembles markedly that in the Simulation 2.1. However, periods between amplitudes of share prices have shortened. First autocorrelation coefficients are again highly significant but they decay more quickly than in the Simulation 2.1. This is caused by the presence of mean-reversion traders (who "act" against the mean-variance traders) and relative chartists (whose behaviour, as stated before, is on average random). Another change is the long memory of absolute returns, which has become shorter. Kurtosis is still very high, although it has decreased on average if we compare it with Simulation 2.1. It is most probably caused again by mean reversion traders, who hamper the prices to deviate much from their mean values.

4.4 Simulation 4 - IES Market 1

We have seen in the previous simulation that behaviour of stock prices is not very realistic if we mix all four types of traders in the market. The aim of modifications in the IES market 1 and 2 is to mitigate these unfavourable features (highly significant autocorrelations of returns and "too long" memory of absolute returns). In both adjustments, a ratio of number of noise to other traders, is more realistic in comparison with Simulation 3. In the IES Market 1, we have simulated prices for a set-up, where 1.42 noise traders can trade with each "rational trader". In the IES Market 2, this ratio is even better - each "rational" trader can find 1.33 noise traders to trade with. This number was 1.66 in the Simulation 3.

In the first case, the first flaw (autocorrelation of raw returns) was almost removed. Almost all autocorrelation coefficients are insignificant, except of the first one. But this is not a very serious problem, since returns in very short periods can be also significant and as LeBaron points out, we do not know how one period in agent-based market is long. In addition, the first autocorrelation coefficient is negative and it reflects the mean-reverting behaviour of share prices. Another favourable improvement is that long memory has "shortened" in comparison with Simulation 3.

This simulation has however one major drawback. Kurtosis has decreased significantly, although excess kurtosis remained positive in all cases and normality was rejected everywhere as well.

4.5 Simulation 5 - IES Market 2

The second adjustment of the market provides a bit worse results. A few first autocorrelations of raw returns are negatively significant, albeit with only small coefficients. Kurtosis of distributions of returns are even smaller and memory of absolute returns even shorter. Nevertheless, non-normality of returns and volatility clustering were detected in all cases again.

4.6 Interdependence of asset returns

It is also interesting to see how dependence of returns among assets changes among simulations. We will assess interdependence among assets by three measures. The first one is the standard Pearson's correlation coefficient. Its use is sufficient in cases of elliptical families of distributions (Jondeau *et al.* (2007, p.242)). However, when distributions are characterised by fat tails or other non-normalities, which is the case of distributions of asset returns, Kendal's tau and Spearman's rho measures are often used. They have similar favourable properties as the Pearson's correlation coefficient. They range between -1 and 1, coefficient of magnitude 0 mean no dependence (although it does not imply independence in the statistical sense). More on their computations and properties is given in Jondeau *et al.* (2007).

The estimated correlation matrices of asset returns are provided in the Appendix in Table A.1. As can be seen in the table, correlation coefficients are positive and very low in Simulation 1 and Simulation 2.3. This is probably caused by two facts given by cash constraint and random behaviour of traders. It can happen that the amount of cash invested in assets increases/decreases (this is given by behaviour of noise traders). If this happens, noise traders do not care in which asset to invest and they invest evenly in each asset on average. This causes correlation coefficient be positive. However, if amount of cash was constant in the market, the correlation coefficients would be negative (when more money is invested in asset A, less money is invested in asset B and vice versa). This mitigates the first effect, so finally the correlation is positive but very small.

In Simulation 2.1 and 2.2., the situation is a bit different because the first effect does not hold here. When cash flows in the market (as a result of changes in noise traders' behaviour), this cash is not spread evenly in assets and therefore their returns are not correlated positively as in Simulation 1 and 2.3. The second effect, that all assets usually cannot rise at the same time, only reinforces the first one. But since a large proportion of traders' behaviour is random, these correlations are not very large in absolute value.

In Simulation 3, forces that cause correlations be negative in Simulations 2.1 and 2.2 outweigh those that cause them to be positive in Simulation 1 and 2.3. An interesting fact is that correlations are higher (in absolute value) if we increase time lags for which we compute returns.

This holds also for IES Market 1 and IES Market 2. The results are given again in Table A.1 and figures in Section A.9 and A.10. In IES Market 1, correlations of "first order" returns (i.e. returns defined as $\log(r_t/r_{t-1})$are negative. But as we increase the lag with which we compute the returns (e.g. $\log(r_t/r_{t-3})$), the correlation coefficients become positive and all of them increase. It has a simple explanation - inflow/outflow of the cash in/out of the market does not hit all the assets equally at once, but it takes some, albeit short, time to take full effect.

The only difference between the IES Market 1 and 2 as far as interdependence of assets is concerned is that correlation coefficients are positive for returns of the "first order" in the IES Market 2 whilst they are negative in the IES Market 1. But again, as we increase the lag in returns, correlations become higher (and positive) in both cases and they are even higher in the IES Market 2. This is caused by some persistence of changes in cash held in risky assets, which does not exist in the IES Market 1.

The interdependence of this kind might be one of the explanations of financial contagion (or financial crises) - a fall in prices of one share causes falls in prices of other shares, even if companies of these shares are "healthy". For example, Feldman (2008) regards this effect as the only source of crises. However, there are no fundamental variables that could distinguish shares of various companies, so all of them are affected equally. In the final chapter, we propose some steps that could improve the results.

Chapter 5

Conclusions

Agent-based modelling is a promising methodology to model financial and other economic phenomena. In this thesis, we have shown quite a simple model of an artificial stock market which can replicate some of the stylized facts of financial markets under specific sets of initial parameters.

According to the model, stylized facts are caused by irrationality rather than by rationality. The best results are obtained when behaviour of traders is random and when it changes very quickly. When some restrictions on this random behaviour are imposed (traders still decide randomly but a range of a random parameter is narrowed), the stylized facts persist but are smaller in magnitude.

When more sophisticated traders are added to the market, some of the stylized facts remain but some unfavourable properties emerge. Returns on shares become autocorrelated and shapes of price paths of shares begin to deviate from those observed in reality. These drawbacks were partially resolved by two modifications, when agents decide on the amount of cash they invest in shares according to the performance of portfolios of traders in the market. However, the other stylized facts become weaker as a result of these modifications.

The two proposed and implemented modifications have one additional contribution. They bring about interdependence of asset returns to the market and the returns become correlated as a result. This could be a first step to implement a more realistic artificial stock market based on the GASM model. However, magnitudes of share returns correlations are similar among all shares and this must change by an introduction of other modifications.

The main feature which we think should be implemented in the model is the use of some fundamental variables of companies and of the market as a whole. In the current version of the model, share prices move only as a result of random behaviour of traders or behaviour of traders who investigate past price paths. If fundamental variables, such as dividends, profits of firms etc. were present, much greater variety of trading strategies would be possible. It would be very interesting to see whether the stylized facts of financial markets would remain in this version of the market. In addition, this would make correlations between various types of shares significantly different, not as in the case of the IES Market 1 or 2.

Another improvement is related again to the fundamental variables. No interest rates have been mentioned so far. Adding interest rates would enable us to produce a greater scope for improvements of the market. First, interest rates would be incorporated in portfolio decisions of the traders. They could borrow funds in order to buy more shares. Or they could deposit money when they believed that this would provide them with higher returns. In addition, it would be possible to make simple experiments of how monetary policy can affect stock markets. Finally, interest rates would play a role when more financial markets would be added to the model, along with exchange rates. It would be possible to observe for example, what role interest rates play in spreading of financial crises from one market to the other.

Next thing that would improve characteristics of time series of returns and prices, most importantly eliminate autocorrelations of raw returns, would be to add more sophisticated chartist traders. At the beginning, some relatively simple versions would be sufficient. In later stages, it would be interesting to employ traders similar to those used in the Santa Fe Artificial Stock Market, who are endowed with learning abilities and whose behaviour can recognize plenitude of market conditions that can arise.

Finally, the main framework of the model should be slightly adjusted to prevent the flaw mentioned in Section 3.1 that causes investors to lack money in some cases. This would be partly resolved by allowing leveraging - when traders do not have money to buy shares, they simply borrow it. But this is only a partial solutions, since they did not intend to borrow money, they were just forced to do so. So an improvement about timing of processing of limit orders would be appropriate.

It will not be easy to implement all these changes. But we believe that if more researchers would cooperate on this project, the solution would be feasible.

Appendix A

Simulation results

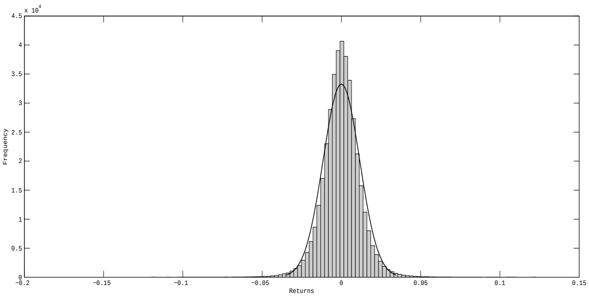
A.1 Simulation 1

Market setup

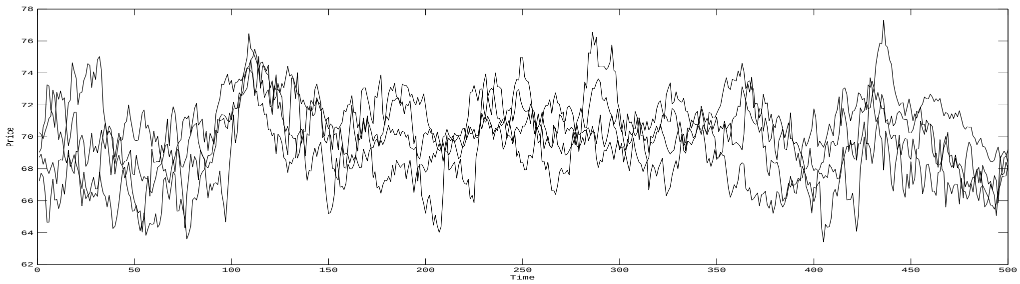
of traders/probability

Noise traders: 10,000 / 0.01

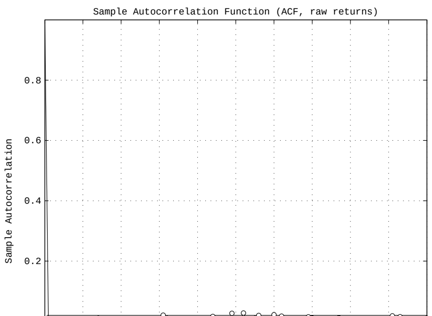
Histogram of asset returns

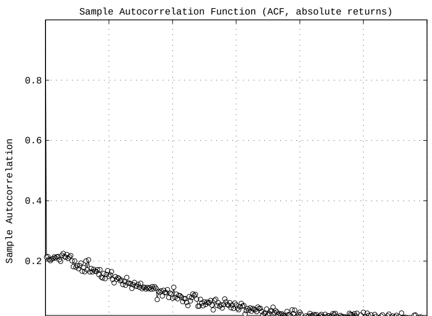


Typical stock price evolution



Autocorrelation functions





Statistics of returns

	min	max	mean	std		min	max	mean	std
Min	-0.120	-0.053	-0.077	0.016	JB Stat	586.80	14469.68	3454.41	3122.67
Max	0.046	0.123	0.076	0.018	Critical value	5.9817			
Mean	-1.177E-05	8.240E-06	-1.012E-06	4.998E-06	LBQ stat	65.06	178.88	105.14	22.51
std. dev	0.01000	0.01268	0.01152	0.00074	Critical value	31.4104			
studentized range	9.827	19.639	13.225	2.146	Arch test	175.85	890.22	428.94	193.04
Kurtosis	4.184	8.893	5.643	1.150	Critical value	3.8415			
Skewness	-0.149	0.150	-0.006	0.056	Long memory	65	521	148.175	86.45582465

A.1.1 Noise market - adjustment 1

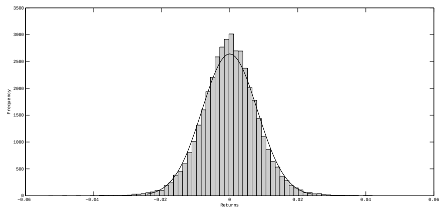
Market setup

\# of traders/probability

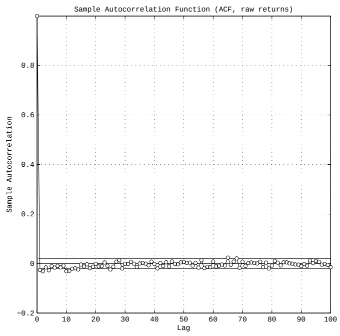
Noise traders: 10,000 / 0.01

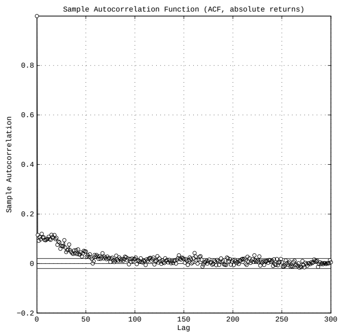
$\hat{\gamma}_i \sim U(0.3, 0.7)$

Histogram of asset returns



Autocorrelation functions





A.1.2 Noise market - adjustment 2

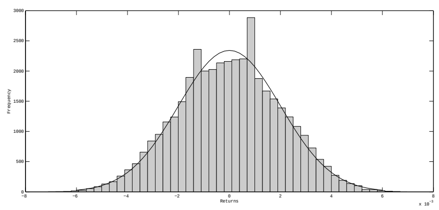
Market setup

\# of traders/probability

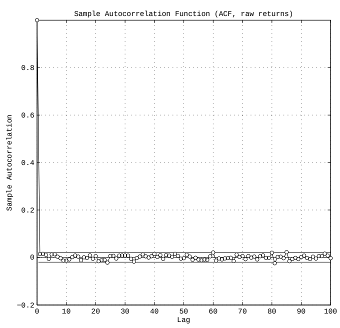
Noise traders: 10,000 / 0.1

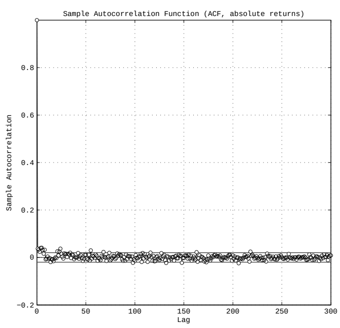
$\hat{\gamma}_i \sim U(0, 1)$

Histogram of asset returns



Autocorrelation functions





A.2 Simulation 2.1

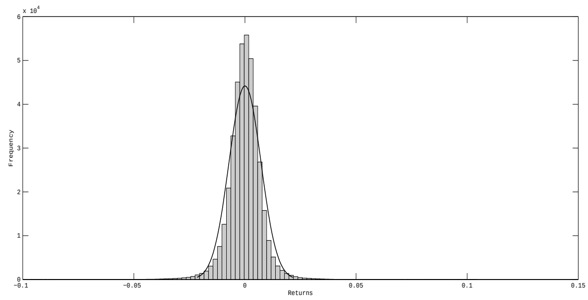
Market setup

of traders/probability

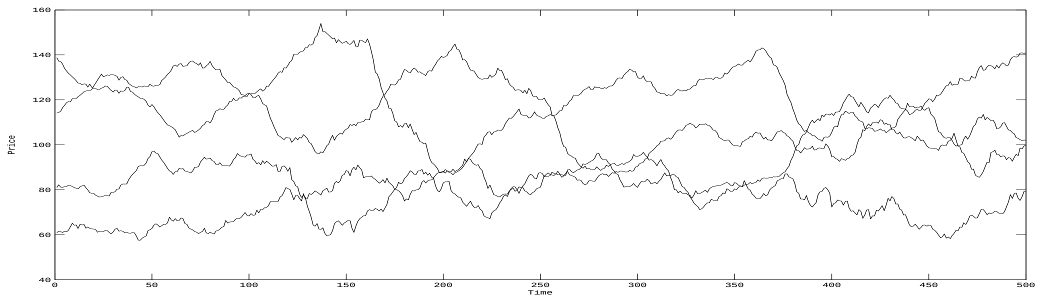
Noise traders: 1,000 / 0.1

Mean-variance t.: 1,000 / 0.1

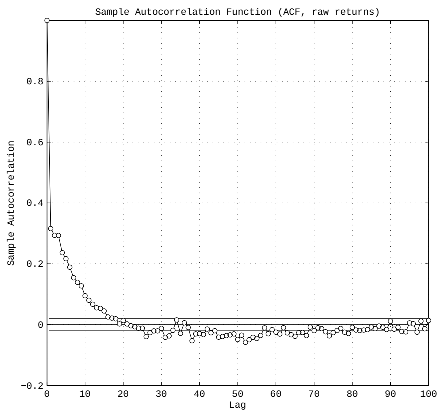
Histogram of asset returns

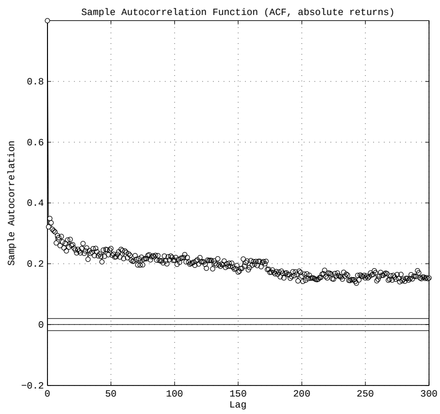


Typical stock price evolution



Autocorrelation functions





Statistics of returns

	min	max	mean	std		min	max	mean	std
Min	-0.0968	-0.0380	-0.0619	0.0155	JB Stat	1 212	74 163	17 020	17 295
Max	0.0321	0.1042	0.0557	0.0160	Critical value	5.9817			
Mean	-0.000033	0.000055	0.000004	0.000022	LBQ stat	2407.29	8210.49	5250.86	1276.68
std. dev	0.0060	0.0085	0.0072	0.0005	Critical value	31.4104			
studentized range	10.6449	23.6951	16.3066	3.2039	Arch test	323.7756	1380.4433	819.9985	239.2902
Kurtosis	4.6722	16.3161	8.6712	2.9259	Critical value	3.8415			
Skewness	-0.7498	-0.0342	-0.2520	0.1473	Long memory	1514	2894	2173	269.81

A.3 Simulation 2.2

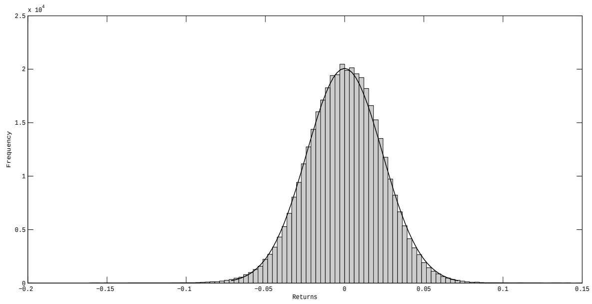
Market setup

of traders/probability

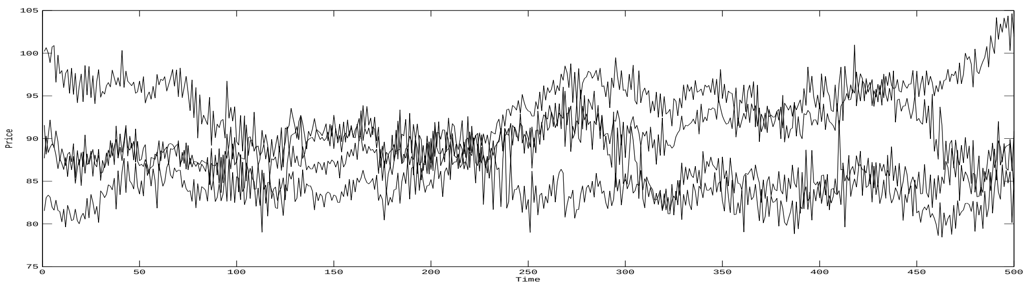
Noise traders: 1,000 / 0.1

Mean-reverting t.: 1,000 / 0.1

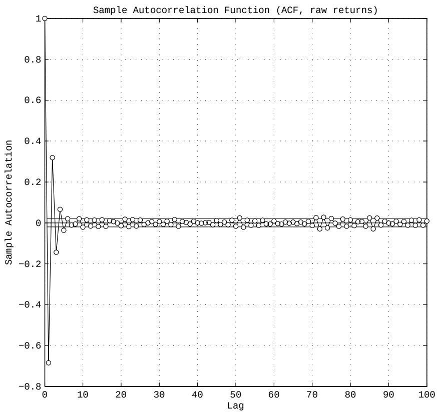
Histogram of asset returns

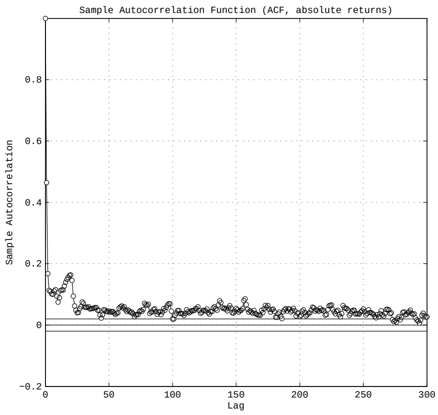


Typical stock price evolution



Autocorrelation functions





Statistics of returns

	min	max	mean	std		min	max	mean	std
Min	-0.161	-0.087	-0.115	0.015	JB Stat	13.96	239.77	78.05	51.78
Max	0.084	0.143	0.105	0.013	Critical value	5.9817			
Mean	-2.776E-05	6.900E-06	-1.058E-05	8.942E-06	LBQ stat	5444.82	6344.87	5819.77	208.52
std. dev	0.02174	0.02566	0.02388	0.00073	Critical value	31.4104			
studentized range	7.545	12.515	9.205	0.957	Arch test	1778.27	2376.72	2045.27	138.31
Kurtosis	2.955	3.740	3.355	0.165	Critical value	3.8415			
Skewness	-0.134	-0.033	-0.090	0.023	Long memory	24.00	452.00	91.65	100.19

A.4 Simulation 2.3

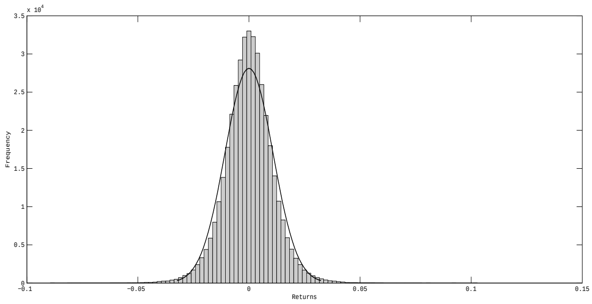
Market setup

of traders/probability

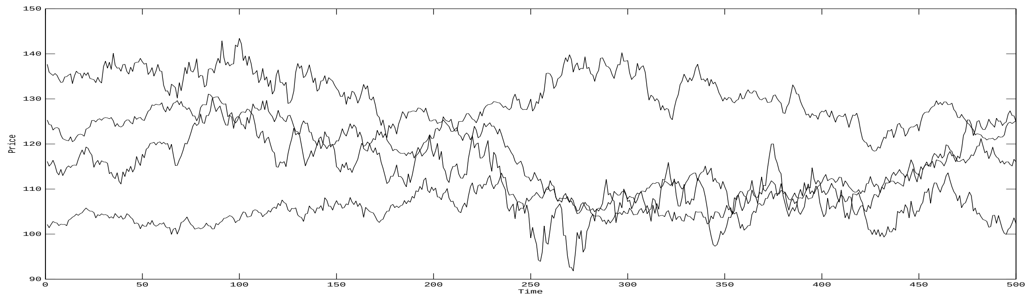
Noise traders: 1,000 / 0.1

Relative chartists.: 1,000 / 0.1

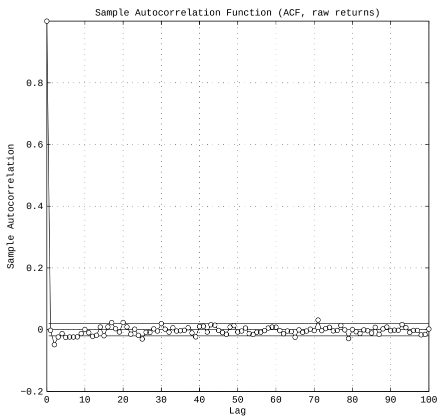
Histogram of asset returns

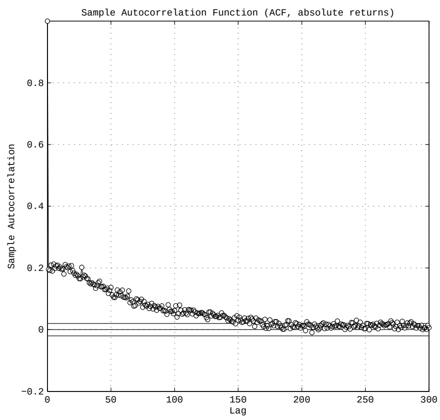


Typical stock price evolution



Autocorrelation functions





Statistics of returns

	min	max	mean	std		min	max	mean	std
Min	-0.089	-0.041	-0.064	0.011	JB Stat	314.00	3535.89	1390.36	885.95
Max	0.042	0.103	0.064	0.013	Critical value	5.9817			
Mean	-3.867E-05	4.929E-05	-1.143E-06	2.168E-05	LBQ stat	31.43	105.79	58.97	16.88
std. dev	0.00965	0.01313	0.01078	0.00080	Critical value	31.4104			
studentized range	9.172	15.128	11.806	1.452	Arch test	115.99	743.98	319.08	120.43
Kurtosis	3.868	5.913	4.736	0.568	Critical value	3.8415			
Skewness	-0.104	0.107	-0.015	0.048	Long memory	63.00	196.00	114.78	38.10

A.5 Simulation 3

Market setup

of traders/probability

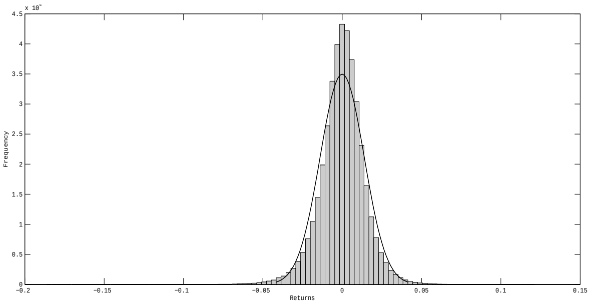
Noise traders: 1,500 / 0.1

Mean-variance t.: 300 / 0.1

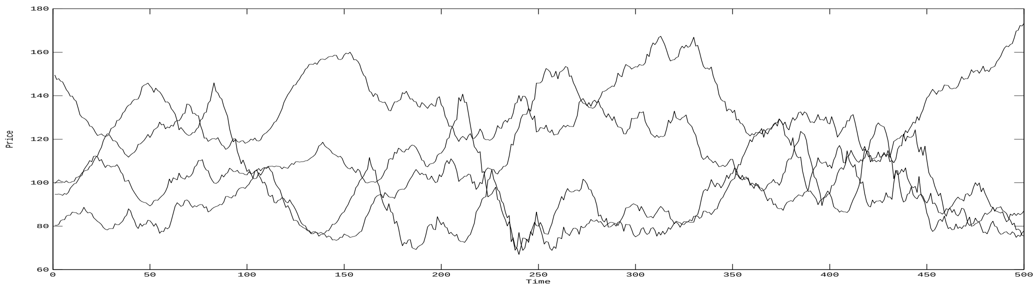
Mean-reverting t.: 300 / 0.1

Relative chartists.: 300 / 0.1

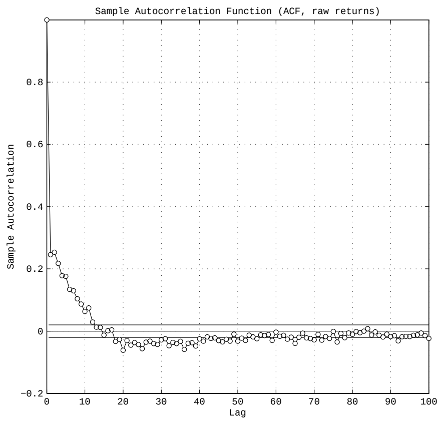
Histogram of asset returns

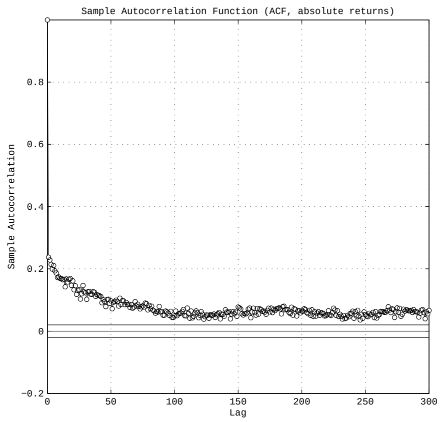


Typical stock price evolution



Autocorrelation functions





Statistics of returns

	min	max	mean	std		min	max	mean	std
Min	-0.186	-0.071	-0.104	0.026	JB Stat	960.83	76297.13	5650.09	12039.76
Max	0.059	0.121	0.083	0.015	Critical value	5.9817			
Mean	-5.001E-05	2.314E-05	-1.060E-05	1.893E-05	LBQ stat	2110.48	4624.34	3264.96	610.23
std. dev	0.01320	0.01487	0.01389	0.00041	Critical value	31.4104			
studentized range	10.369	19.732	13.431	2.121	Arch test	258.50	2605.14	593.40	402.23
Kurtosis	4.411	16.269	5.998	2.004	Critical value	3.8415			
Skewness	-1.328	-0.126	-0.347	0.210	Long memory	61.00	2605.14	393.00	330.27

A.6 IES Market 1

Market setup

\# of traders/probability

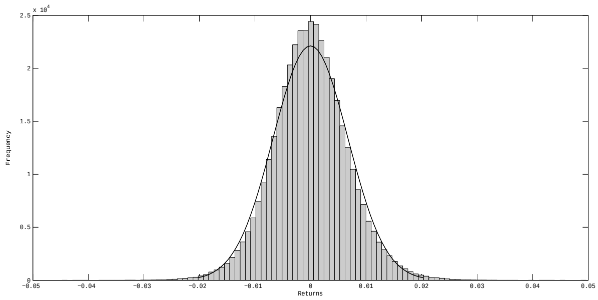
Noise traders: 1,700 / 0.1

Mean-variance t.: 400 / 0.1

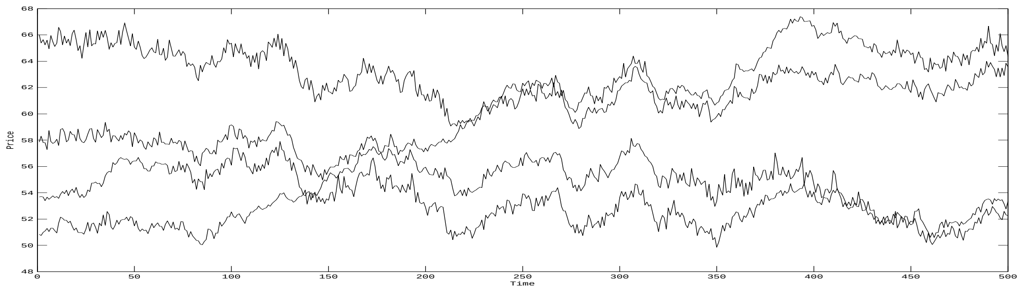
Mean-reverting t.: 400 / 0.1

Relative chartists.: 400 / 0.1

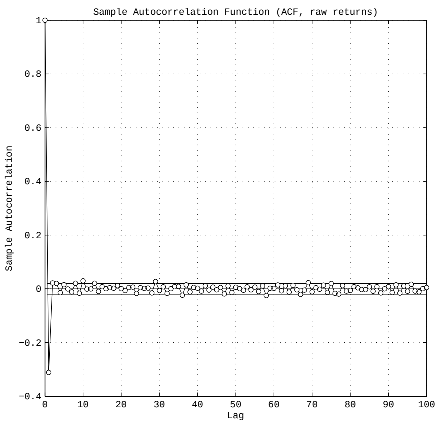
Histogram of asset returns

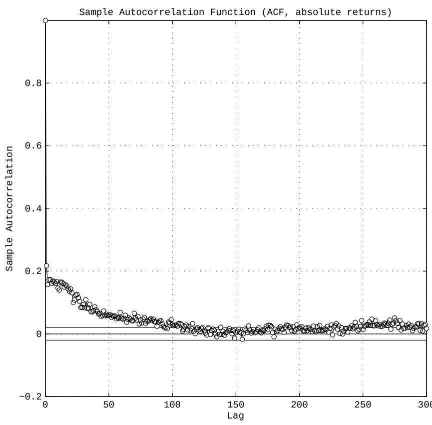


Typical stock price evolution



Autocorrelation functions





Statistics of returns

	min	max	mean	std		min	max	mean	std
Min	-0.045	-0.025	-0.034	0.004	JB Stat	127.07	708.07	331.99	156.56
Max	0.028	0.050	0.035	0.005	Critical value	5.9817			
Mean	-3.634E-05	3.760E-05	-2.046E-06	1.379E-05	LBQ stat	530.46	1288.62	922.88	165.99
std. dev	0.00598	0.00749	0.00673	0.00035	Critical value	31.4104			
studentized range	7.917	13.279	10.290	1.132	Arch test	253.19	670.86	438.96	117.73
Kurtosis	3.446	4.299	3.846	0.219	Critical value	3.8415			
Skewness	-0.018	0.163	0.081	0.045	Long memory	63.00	1287.00	175.35	226.63

A.7 IES Market 2

Market setup

of traders/probability

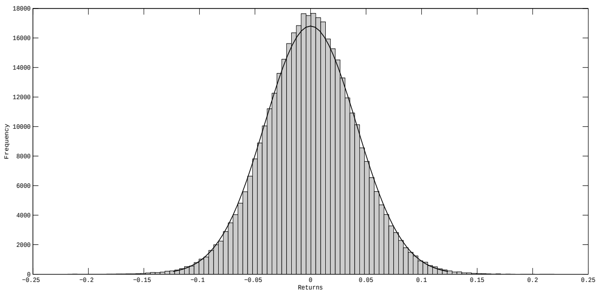
Noise traders: 1,600 / 0.1

Mean-variance t.: 400 / 0.1

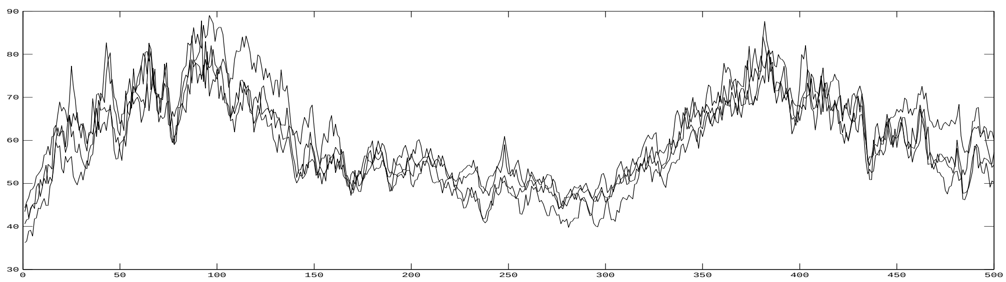
Mean-reverting t.: 400 / 0.1

Relative chartists.: 400 / 0.1

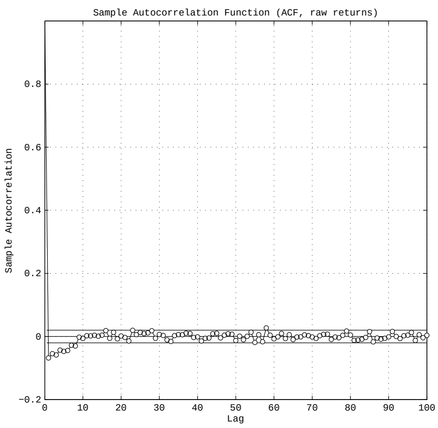
Histogram of asset returns

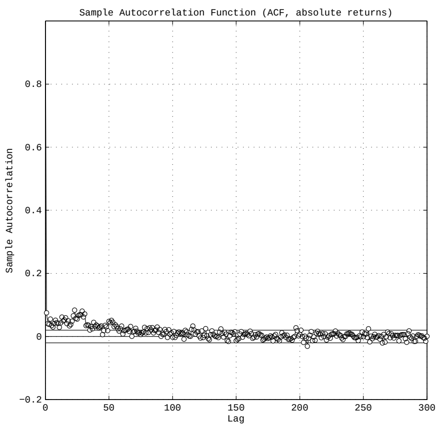


Typical stock price evolution



Autocorrelation functions





Statistics of returns

	min	max	mean	std		min	max	mean	std
Min	-0.219	-0.153	-0.184	0.021	JB Stat	7.44	128.66	53.22	30.19
Max	0.151	0.219	0.178	0.017	Critical value	5.9817			
Mean	-4.427E-05	3.741E-05	4.525E-06	1.873E-05	LBQ stat	191.79	269.63	221.80	17.96
std. dev	0.03859	0.04390	0.04117	0.00118	Critical value	31.4104			
studentized range	7.624	10.090	8.808	0.612	Arch test	13.60	137.40	55.66	30.40
Kurtosis	3.128	3.537	3.340	0.097	Critical value	3.8415			
Skewness	-0.072	0.053	-0.008	0.028	Long memory	4.00	74.00	50.68	19.10

A.8 Interdependence of asset returns

Table A.1: Correlation matrices of asset returns

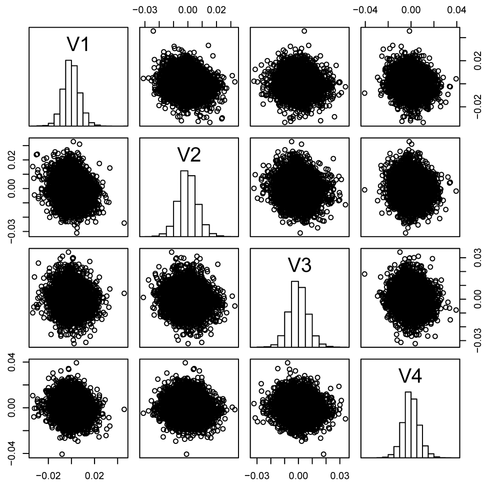
Sim/Lag	Pearson's r				Kendall's τ				Spearman's ρ			
Noise												
1	1.000	0.017	0.020	0.004	1.000	0.011	0.012	0.012	1.000	0.017	0.018	0.019
	0.017	1.000	0.016	0.020	0.011	1.000	0.014	0.017	0.017	1.000	0.021	0.025
	0.020	0.016	1.000	0.021	0.012	0.014	1.000	0.013	0.018	0.021	1.000	0.020
	0.004	0.020	0.021	1.000	0.012	0.017	0.013	1.000	0.019	0.025	0.020	1.000
Sim 2.1												
1	1.000	-0.117	-0.126	-0.122	1.000	-0.082	-0.074	-0.086	1.000	-0.120	-0.109	-0.125
	-0.117	1.000	-0.079	-0.081	-0.082	1.000	-0.077	-0.070	-0.120	1.000	-0.111	-0.101
	-0.126	-0.079	1.000	-0.098	-0.074	-0.077	1.000	-0.082	-0.109	-0.111	1.000	-0.119
	-0.122	-0.081	-0.098	1.000	-0.086	-0.070	-0.082	1.000	-0.125	-0.101	-0.119	1.000
Sim 2.2												
1	1.000	-0.188	-0.193	-0.213	1.000	-0.134	-0.136	-0.151	1.000	-0.198	-0.201	-0.223
	-0.188	1.000	-0.197	-0.204	-0.134	1.000	-0.137	-0.142	-0.198	1.000	-0.204	-0.212
	-0.193	-0.197	1.000	-0.233	-0.136	-0.137	1.000	-0.161	-0.201	-0.204	1.000	-0.238
	-0.213	-0.204	-0.233	1.000	-0.151	-0.142	-0.161	1.000	-0.223	-0.212	-0.238	1.000
Sim 2.3												
1	1.000	0.011	0.018	0.009	1.000	0.006	0.016	0.004	1.000	0.009	0.024	0.006
	0.011	1.000	0.024	0.012	0.006	1.000	0.016	0.008	0.009	1.000	0.024	0.013
	0.018	0.024	1.000	0.010	0.016	0.016	1.000	0.011	0.024	0.024	1.000	0.016
	0.009	0.012	0.010	1.000	0.004	0.008	0.011	1.000	0.006	0.013	0.016	1.000
Sim 3												
1	1.000	-0.120	-0.107	-0.128	1.000	-0.084	-0.070	-0.084	1.000	-0.124	-0.104	-0.125
	-0.120	1.000	-0.114	-0.143	-0.084	1.000	-0.074	-0.102	-0.124	1.000	-0.109	-0.151
	-0.107	-0.114	1.000	-0.103	-0.070	-0.074	1.000	-0.070	-0.104	-0.109	1.000	-0.103
	-0.128	-0.143	-0.103	1.000	-0.084	-0.102	-0.070	1.000	-0.125	-0.151	-0.103	1.000
3	1.000	-0.207	-0.160	-0.181	1.000	-0.144	-0.103	-0.121	1.000	-0.212	-0.152	-0.177
	-0.207	1.000	-0.170	-0.220	-0.144	1.000	-0.097	-0.154	-0.212	1.000	-0.143	-0.227
	-0.160	-0.170	1.000	-0.162	-0.103	-0.097	1.000	-0.114	-0.152	-0.143	1.000	-0.167
	-0.181	-0.220	-0.162	1.000	-0.121	-0.154	-0.114	1.000	-0.177	-0.227	-0.167	1.000
4	1.000	-0.218	-0.169	-0.210	1.000	-0.150	-0.110	-0.140	1.000	-0.222	-0.162	-0.204
	-0.218	1.000	-0.188	-0.251	-0.150	1.000	-0.107	-0.171	-0.222	1.000	-0.157	-0.250
	-0.169	-0.188	1.000	-0.183	-0.110	-0.107	1.000	-0.120	-0.162	-0.157	1.000	-0.177
	-0.210	-0.251	-0.183	1.000	-0.140	-0.171	-0.120	1.000	-0.204	-0.250	-0.177	1.000
5	1.000	-0.242	-0.177	-0.218	1.000	-0.158	-0.114	-0.134	1.000	-0.233	-0.169	-0.198
	-0.242	1.000	-0.198	-0.266	-0.158	1.000	-0.112	-0.190	-0.233	1.000	-0.165	-0.278
	-0.177	-0.198	1.000	-0.193	-0.114	-0.112	1.000	-0.119	-0.169	-0.165	1.000	-0.175
	-0.218	-0.266	-0.193	1.000	-0.134	-0.190	-0.119	1.000	-0.198	-0.278	-0.175	1.000
6	1.000	-0.258	-0.196	-0.211	1.000	-0.177	-0.130	-0.140	1.000	-0.261	-0.192	-0.205
	-0.258	1.000	-0.199	-0.286	-0.177	1.000	-0.110	-0.197	-0.261	1.000	-0.164	-0.288
	-0.196	-0.199	1.000	-0.190	-0.130	-0.110	1.000	-0.126	-0.192	-0.164	1.000	-0.184
	-0.211	-0.286	-0.190	1.000	-0.140	-0.197	-0.126	1.000	-0.205	-0.288	-0.184	1.000
7	1.000	-0.263	-0.200	-0.242	1.000	-0.171	-0.134	-0.160	1.000	-0.253	-0.197	-0.235
	-0.263	1.000	-0.226	-0.282	-0.171	1.000	-0.125	-0.200	-0.253	1.000	-0.186	-0.293
	-0.200	-0.226	1.000	-0.197	-0.134	-0.125	1.000	-0.122	-0.197	-0.186	1.000	-0.179
	-0.242	-0.282	-0.197	1.000	-0.160	-0.200	-0.122	1.000	-0.235	-0.293	-0.179	1.000
IESM1												
1	1.000	-0.136	-0.071	-0.080	1.000	-0.083	-0.040	-0.039	1.000	-0.123	-0.060	-0.059
	-0.136	1.000	-0.110	-0.097	-0.083	1.000	-0.070	-0.063	-0.123	1.000	-0.105	-0.094
	-0.071	-0.110	1.000	-0.091	-0.040	-0.070	1.000	-0.054	-0.060	-0.105	1.000	-0.081
	-0.080	-0.097	-0.091	1.000	-0.039	-0.063	-0.054	1.000	-0.059	-0.094	-0.081	1.000
2	1.000	0.075	0.105	0.121	1.000	0.045	0.067	0.081	1.000	0.067	0.100	0.120
	0.075	1.000	0.089	0.106	0.045	1.000	0.058	0.065	0.067	1.000	0.086	0.097
	0.105	0.089	1.000	0.112	0.067	0.058	1.000	0.065	0.100	0.086	1.000	0.097
	0.121	0.106	0.112	1.000	0.081	0.065	0.065	1.000	0.120	0.097	0.097	1.000
3	1.000	0.224	0.238	0.190	1.000	0.145	0.144	0.129	1.000	0.215	0.214	0.191
	0.224	1.000	0.194	0.228	0.145	1.000	0.122	0.135	0.215	1.000	0.183	0.200
	0.238	0.194	1.000	0.221	0.144	0.122	1.000	0.138	0.214	0.183	1.000	0.204
	0.190	0.228	0.221	1.000	0.129	0.135	0.138	1.000	0.191	0.200	0.204	1.000
4	1.000	0.338	0.315	0.321	1.000	0.215	0.196	0.199	1.000	0.316	0.289	0.292
	0.338	1.000	0.286	0.286	0.215	1.000	0.184	0.179	0.316	1.000	0.272	0.263
	0.315	0.286	1.000	0.332	0.196	0.184	1.000	0.203	0.289	0.272	1.000	0.298
	0.321	0.286	0.332	1.000	0.199	0.179	0.203	1.000	0.292	0.263	0.298	1.000
5	1.000	0.365	0.391	0.386	1.000	0.223	0.256	0.246	1.000	0.327	0.371	0.358
	0.365	1.000	0.334	0.376	0.223	1.000	0.212	0.238	0.327	1.000	0.309	0.348
	0.391	0.334	1.000	0.376	0.256	0.212	1.000	0.237	0.371	0.309	1.000	0.346
	0.386	0.376	0.376	1.000	0.246	0.238	0.237	1.000	0.358	0.348	0.346	1.000

Continued on Next Page…

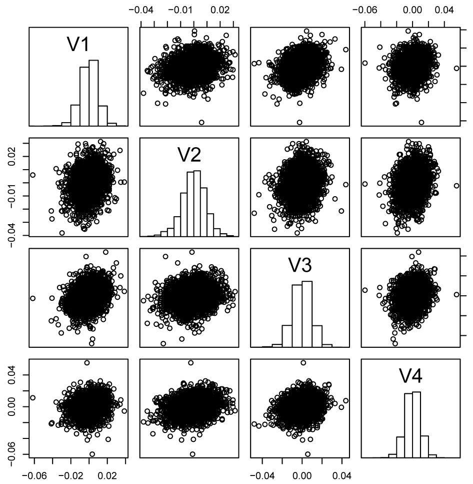
Table A.1 – Continued

Sim/Lag	Pearson's r				Kendall's τ				Spearman's ρ			
6	1.000	0.420	0.392	0.393	1.000	0.270	0.258	0.265	1.000	0.393	0.377	0.384
	0.420	1.000	0.396	0.419	0.270	1.000	0.267	0.266	0.393	1.000	0.387	0.386
	0.392	0.396	1.000	0.429	0.258	0.267	1.000	0.273	0.377	0.387	1.000	0.397
	0.393	0.419	0.429	1.000	0.265	0.266	0.273	1.000	0.384	0.386	0.397	1.000
7	1.000	0.401	0.452	0.425	1.000	0.260	0.315	0.281	1.000	0.375	0.450	0.405
	0.401	1.000	0.379	0.441	0.260	1.000	0.252	0.283	0.375	1.000	0.365	0.407
	0.452	0.379	1.000	0.434	0.315	0.252	1.000	0.297	0.450	0.365	1.000	0.424
	0.425	0.441	0.434	1.000	0.281	0.283	0.297	1.000	0.405	0.407	0.424	1.000
IESM2												
1	1.000	0.015	0.021	0.022	1.000	0.010	0.017	0.017	1.000	0.015	0.025	0.025
	0.015	1.000	0.006	0.032	0.010	1.000	0.004	0.025	0.015	1.000	0.006	0.037
	0.021	0.006	1.000	0.025	0.017	0.004	1.000	0.016	0.025	0.006	1.000	0.024
	0.022	0.032	0.025	1.000	0.017	0.025	0.016	1.000	0.025	0.037	0.024	1.000
2	1.000	0.180	0.176	0.195	1.000	0.116	0.110	0.122	1.000	0.173	0.164	0.182
	0.180	1.000	0.188	0.185	0.116	1.000	0.122	0.120	0.173	1.000	0.182	0.179
	0.176	0.188	1.000	0.227	0.110	0.122	1.000	0.150	0.164	0.182	1.000	0.222
	0.195	0.185	0.227	1.000	0.122	0.120	0.150	1.000	0.182	0.179	0.222	1.000
3	1.000	0.286	0.299	0.290	1.000	0.179	0.190	0.184	1.000	0.267	0.282	0.274
	0.286	1.000	0.289	0.319	0.179	1.000	0.185	0.211	0.267	1.000	0.273	0.310
	0.299	0.289	1.000	0.311	0.190	0.185	1.000	0.200	0.282	0.273	1.000	0.294
	0.290	0.319	0.311	1.000	0.184	0.211	0.200	1.000	0.274	0.310	0.294	1.000
4	1.000	0.398	0.369	0.403	1.000	0.254	0.232	0.263	1.000	0.372	0.343	0.384
	0.398	1.000	0.360	0.404	0.254	1.000	0.226	0.262	0.372	1.000	0.331	0.382
	0.369	0.360	1.000	0.409	0.232	0.226	1.000	0.258	0.343	0.331	1.000	0.377
	0.403	0.404	0.409	1.000	0.263	0.262	0.258	1.000	0.384	0.382	0.377	1.000
5	1.000	0.439	0.456	0.462	1.000	0.283	0.299	0.307	1.000	0.411	0.434	0.445
	0.439	1.000	0.408	0.429	0.283	1.000	0.265	0.274	0.411	1.000	0.387	0.398
	0.456	0.408	1.000	0.445	0.299	0.265	1.000	0.289	0.434	0.387	1.000	0.417
	0.462	0.429	0.445	1.000	0.307	0.274	0.289	1.000	0.445	0.398	0.417	1.000
6	1.000	0.476	0.503	0.447	1.000	0.305	0.313	0.289	1.000	0.441	0.454	0.418
	0.476	1.000	0.469	0.488	0.305	1.000	0.300	0.319	0.441	1.000	0.435	0.460
	0.503	0.469	1.000	0.475	0.313	0.300	1.000	0.302	0.454	0.435	1.000	0.437
	0.447	0.488	0.475	1.000	0.289	0.319	0.302	1.000	0.418	0.460	0.437	1.000
7	1.000	0.498	0.513	0.546	1.000	0.324	0.324	0.360	1.000	0.466	0.467	0.517
	0.498	1.000	0.518	0.545	0.324	1.000	0.330	0.351	0.466	1.000	0.474	0.502
	0.513	0.518	1.000	0.533	0.324	0.330	1.000	0.332	0.467	0.474	1.000	0.475
	0.546	0.545	0.533	1.000	0.360	0.351	0.332	1.000	0.517	0.502	0.475	1.000

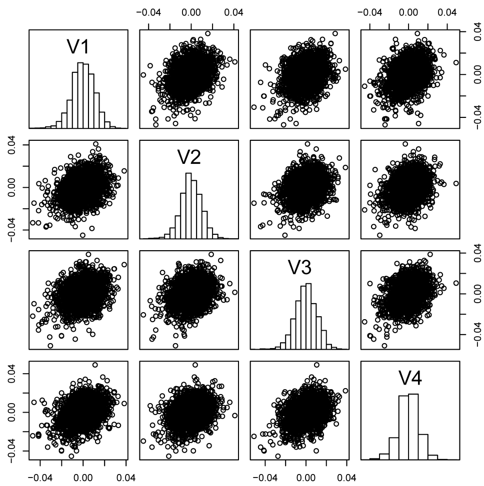
A.9 Interdependence of Returns - IES Market 1



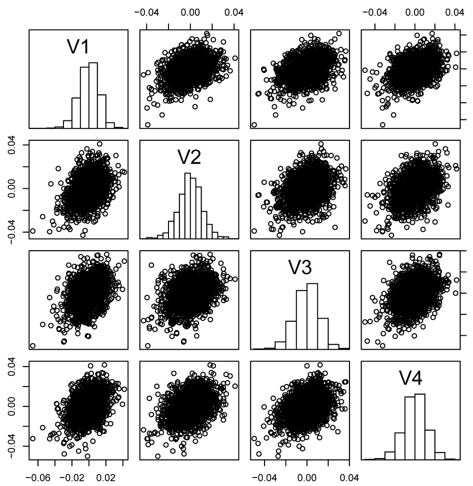
(a) $r_t = \log(p_t - p_{t-1})$



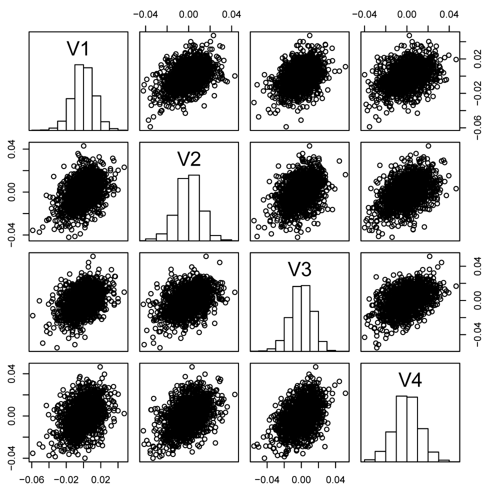
(b) $r_t = \log(p_t - p_{t-2})$



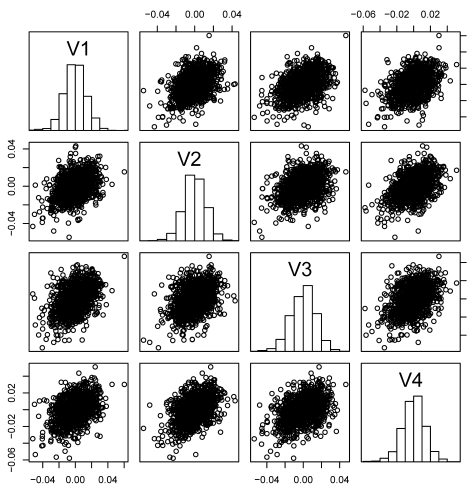
(c) $r_t = \log(p_t - p_{t-3})$



(d) $r_t = \log(p_t - p_{t-4})$

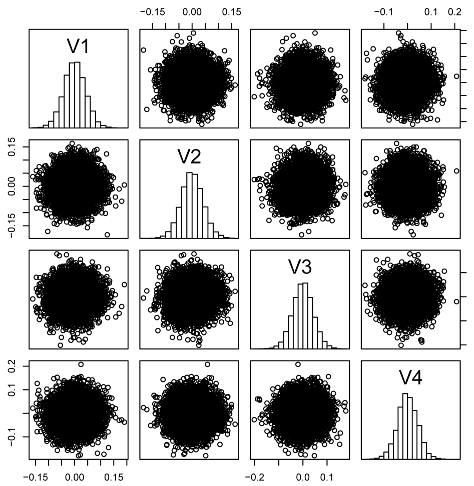


(e) $r_t = \log(p_t - p_{t-5})$

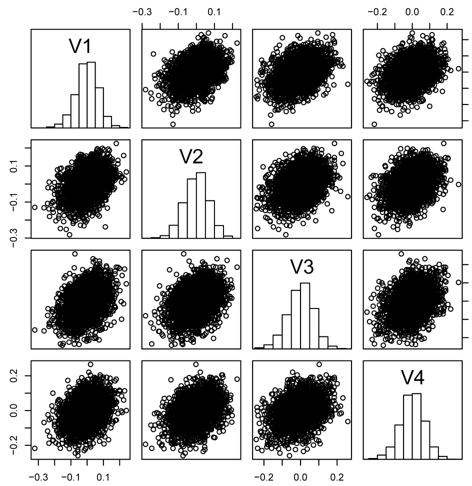


(f) $r_t = \log(p_t - p_{t-6})$

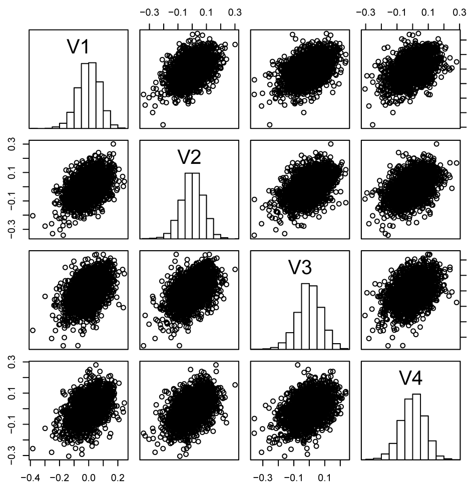
A.10 Interdependence of Returns - IES Market 2



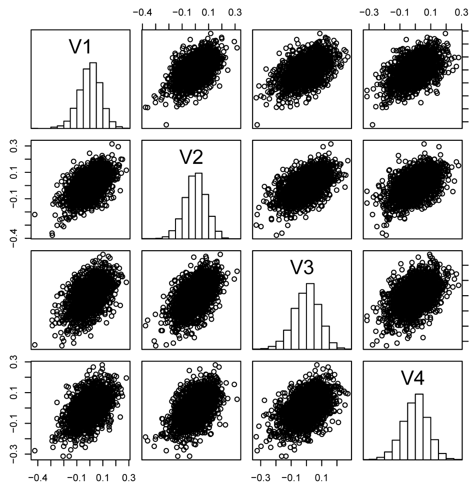
(a) $r_t = \log(p_t - p_{t-1})$



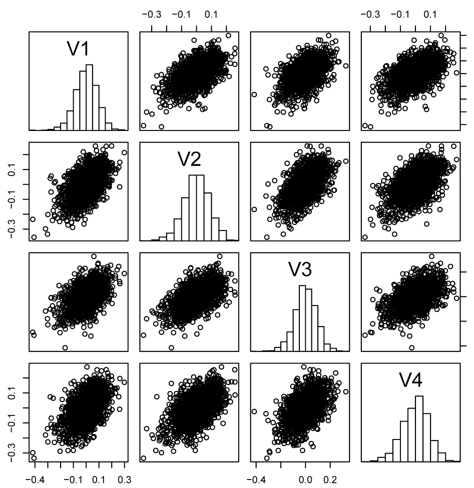
(b) $r_t = \log(p_t - p_{t-3})$



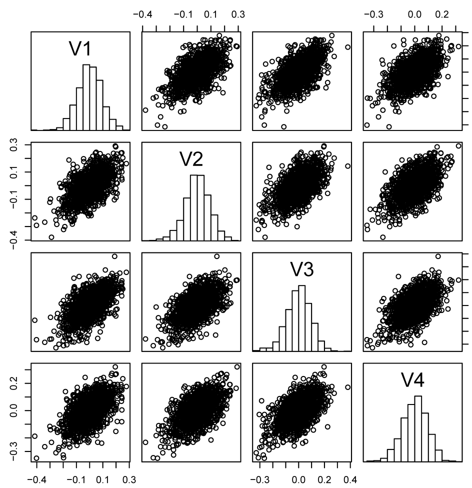
(c) $r_t = \log(p_t - p_{t-4})$



(d) $r_t = \log(p_t - p_{t-5})$



(e) $r_t = \log(p_t - p_{t-6})$



(f) $r_t = \log(p_t - p_{t-7})$

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